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PHYS 211 Exam 1 – Supplement

Circular Motion

Centripetal Acceleration

- There are two ways of defining acceleration we need to be aware of.
- The one we've been using so far deals with linear motion and is typically used when an object's speed changes over a given period of time.
- However, there is another form of acceleration we must consider that results in **circular motion** due to a **change in the direction of velocity**.
- For circular motion, an object can have **an acceleration that causes its direction to constantly change while still maintaining a constant speed**.
- This is referred to as **centripetal acceleration**, and is defined by:

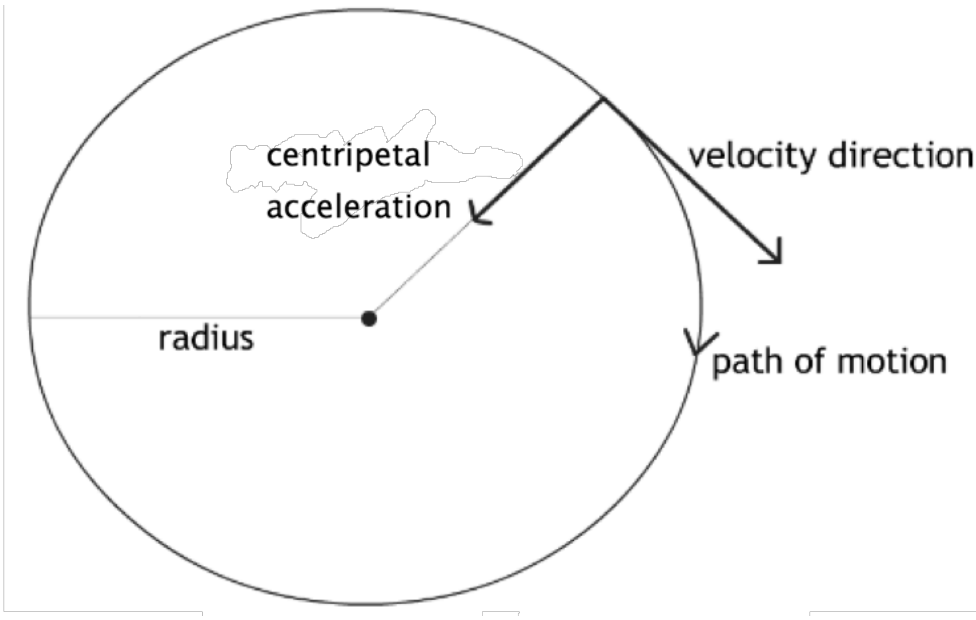
$$a_c = \frac{v^2}{r}$$

where ***v*** is the object's **velocity**, and ***r*** is the **radius** of the circular path.

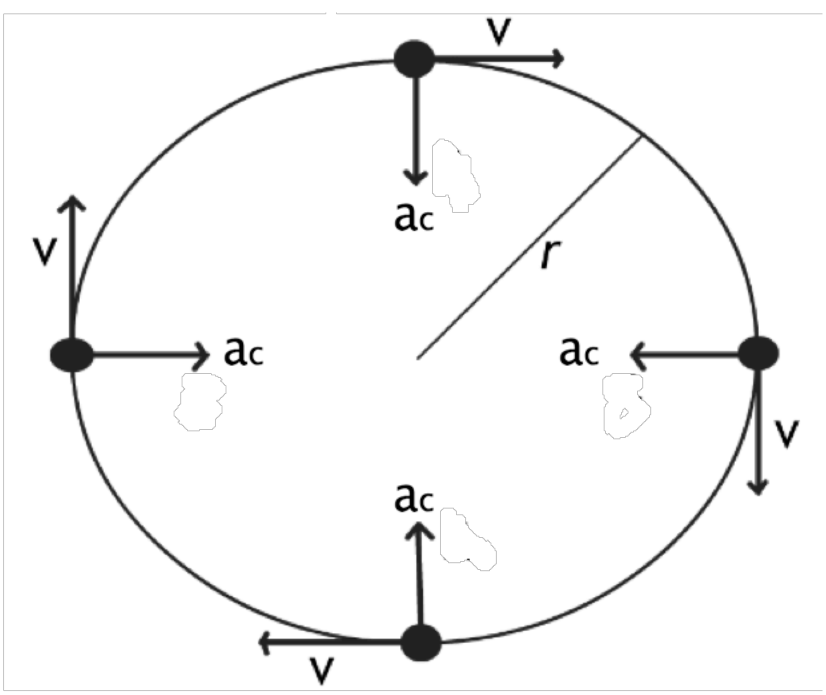
- The **centripetal acceleration**, like all accelerations, is a **vector**, and it always **points towards the center of the circular path**.
- This means that when an object moves in a circular path, it's **velocity will always be tangent to the path**, while its **centripetal acceleration will always be perpendicular to the velocity**, and thus point towards the center.
- As the object moves along the circumference of its circular path, its **velocity and centripetal acceleration will continuously change direction**, however they will always remain perpendicular to one another.

Relating Centripetal Acceleration with Velocity

- When an object moves along a circular path its **velocity will always be tangent to the path**, while its **centripetal acceleration will always be <u>perpendicular</u> to the velocity** and **point towards the center (along the radial axis)**.



- This will always be the relationship between the **centripetal acceleration** and the tangential velocity **at any point along the circular path**, as shown in the clockwise motion below:



Defining Angular Speed

- Continuing with the laws of motion, we introduce rotational motion.
- Now, displacement is measured as a change in **angle** for an object moving in a circular path.
- However, instead of using degrees, we use **radians** (rad).
- To convert from degrees to radians use the following expressions:

$$1\text{ rad} = \frac{360°}{2\pi} = 57.3°$$

$$\theta(rad) = \frac{\pi}{180°}\theta(\text{deg})$$

$$1\text{ revolution} = 2\pi\text{ rad}$$

- **Angular Speed, ω**, is the change in **angle** over change in **time**:

$$\overline{\omega}(rad/s) = \frac{\Delta\theta}{\Delta t}$$

Relating Linear Motion with Angular Speed

- For an object moving in a circular path, we can determine its linear speed in the direction that is tangent to the path.
- This is referred to as the **tangential speed**:

$$v_t = r\omega$$

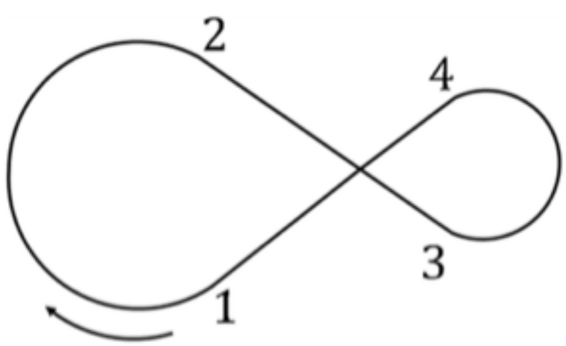
where **r** is the distance from the center of the circular arc to any point on the object

Introducing Frequency and Period

- Any object that moves in a circular path has a **frequency, f,** and **period, T**.
- **Frequency** is defined as the number of cycles (or the number of revolutions) per second, and often has units of Hertz, **Hz, or s^{-1}.**
- **Period** is essentially the inverse of frequency, and is defined as the time it takes to make one revolution, with units of **seconds, s**.
- We can also **relate angular speed, ω, with frequency, f, and period, T**, using the following equation:

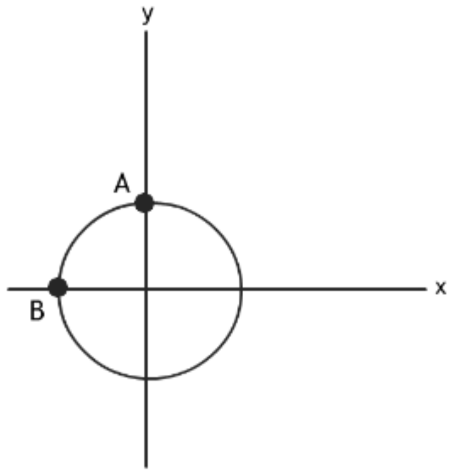
$$\omega = 2\pi f = \frac{2\pi}{T}$$

Problem 1:
A cat moves at a constant speed around a figure-eight track in the direction shown. Where is the magnitude of the object's centripetal acceleration the largest?



A. Between points 1 and 2
B. Between points 2 and 3, and between points 4 and 1
C. Between points 3 and 4
D. It is the same throughout
E. Depends of the magnitude of the constant speed

Problem 2:
A particle moves with constant speed in a circular path, shown below. When the particle is at point A its coordinates are (0, 2) and its velocity is +4i m/s. What are its velocity and acceleration, respectively, at point B?



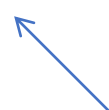
A. -4j m/s, -8i m/s^2
B. +4j m/s, +8i m/s^2
C. -4j m/s, +8i m/s^2
D. +4j m/s, +2i m/s^2
E. +4j m/s, -8i m/s^2

Problem 3:
The dot-motion diagram, shown below, shows the position of an object at five different points in time:

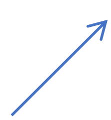
Which of the following vectors best represents the average acceleration vector at time 3?

A. 

B. 

C. 

D. 

E. 

Problem 4:
An unknown bug with incredible suction manages to hang on to the tip of a 0.5-m airplane propeller blade as it rotates at 2 revolutions per second. What is the magnitude of the bug's acceleration?

A. $4\pi^2$ m/s^2
B. $8\pi^2$ m/s^2
C. $16\pi^2$ m/s^2
D. $32\pi^2$ m/s^2
E. $64\pi^2$ m/s^2

Problem 5:
Two cats run in circles around a neighbor's dog, forcing it to stay put. As the two cats move in a circular path, Cat #1 is a distance R from the dog and maintains a constant speed v, while Cat #2 is at a of distance 4R from the dog and maintains a constant speed of $4v$. Given this information, which of the statements below is/are true?

i. Both cats have the same (non-zero) centripetal acceleration.
ii. Both cats have zero acceleration because their speed is constant.
iii. Cat #2 has a centripetal acceleration that is four times that of Cat #1.
iv. Cat #2 completes each circle in one fourth the amount of time it takes Cat #1.
v. Both cats complete each circle in same amount of time.

A. i. and iv.
B. ii. and v.
C. iii. and v.
D. ii. and iv.
E. iii. and iv.

Problem 6:
A record turntable rotates at 45 revolutions per minute. At t = 0, the power to the turntable is shut off and the turntable slows at a constant angular acceleration. Given that the turntable comes to rest after 20 seconds, how many complete rotations did the turntable make during the 20 second interval?

A. 7
B. 8
C. 9
D. 10
E. 11

Solutions
1C

Remember that centripetal acceleration is equal to v^2 / r, and since the magnitude of the velocity (its speed) is constant, the only variable that changes as the cat moves along the figure-eight track is the radius of the circular path.

Between points 2 and 3, as well as between points 4 and 1, the cat moves in a straight line, which means there is no centripetal force.

However, between points 1 and 2, and then later between points 3 and 4, the path becomes circular, with the radius of the circular path between points 1 and 2 being larger than the radius between points 3 and 4. Since radius is on the denominator when calculating centripetal acceleration, the smaller radius will have the larger centripetal acceleration (assuming all else remains constant), and so from points 3 to 4, where the radius is smallest, we have the largest centripetal acceleration.

2B

The velocity at point A is given as +2i, meaning at that instant it's traveling in the +x direction. This would suggest the particle is traveling in a clockwise path, resulting in a velocity that would be point upwards (+y axis) when it reaches point B. And since the speed is constant we keep it at 4 m/s.

The centripetal acceleration is defined as v^2 / r, which becomes $(4)^2 / 2 = 8\ m/s^2$. We get the radius by seeing that the coordinates at point A are 2 units away from the center.

Lastly, the direction of the <u>centripetal</u> acceleration <u>always</u> points towards the <u>center</u> of the circle, which at point B would be towards the right (+x axis).

3B

The dot motion diagram clearly shows the object to be moving in a circular path (or part of a circular path) during the time interval. Circular motion always implies there is some sort of centripetal acceleration causing it to occur, and that vector always points towards the center of the circle. At point 3 the center would be diagonally up and to the left, so the centripetal acceleration would point in that same direction.

However, there is more than one acceleration in play here. If you look closer, the object also appears to be slowing down. This can be seen by observing that the gaps between each time interval get smaller around time 3. Specifically the gap between time 2 and 3 is larger than that of time 3 and 4. If the distance travelled is getting smaller, the speed must be decreasing, which suggests an acceleration in the opposing direction. As a rule, if the speed decreases, there must be an acceleration in the opposite direction causing that to occur. This is acceleration (or deceleration) is NOT the centripetal acceleration, it is a tangential acceleration and points down and to the left since that is the opposite direction of motion at time 3.

Thus we have two accelerations: the centripetal acceleration pointing up and to the left, and the tangential acceleration pointing down and to the left. When we add the two vectors together their upward and downward components cancel, and we get a vector that points towards the left only.

4B

Here we're looking for centripetal acceleration, which only requires velocity and radius. However, we need the velocity in units of m/s, not revolutions per second, so we must convert first. Since one revolution is the exact same distance as the circumference of the circular path, we can use the conversion 1 revolution = 2πr to get:

$$v = \frac{2(2\pi r)}{t} = \frac{2(2\pi \times 0.5)m}{1s} = 2\pi \, m/s$$

And so we get acceleration:

$$a_c = \frac{v_t^2}{r} = \frac{(2\pi)^2}{0.5} = 8\pi^2 \, m/s^2$$

5C

The question is really asking two separate things, what is the centripetal acceleration, and what is the time required to complete one full circle?

To determine centripetal acceleration, we first recognize that it is of course not equal to zero since anytime you move in a circular path there must be a centripetal acceleration. To calculate centripetal acceleration, we use the equation $a_c = v^2/r$, and then plug in the values given for both cats.

For Cat #1 this equals $a_c = v^2/R$,

For Cat #2 this equals $a_c = (4v)^2/(4R) = 16v^2/4R = 4(v^2/R)$

Thus for Cat #2 the centripetal acceleration is four times larger than it is for Cat #1, meaning that Statement iii is correct.

We next want to determine the time it takes to complete one full circle. This time is defined as the period, T, and is related to angular velocity, ω, according to the equation: $\omega = 2\pi / T$.

So in order to determine T, we must first find ω, which we can do using $v = \omega R$, where v is the tangential velocity and R is the radius.

If we put these two equations together and solve for period T, we get:

$$T = 2\pi / \omega = 2\pi / (v/R) = 2\pi R / v$$

Now we determine the periods for both cats:

For Cat #1: $T_1 = 2\pi R/v$

For Cat #2 $T_2 = 2\pi(4R) / (4v) = 2\pi R/v$.

Thus both cats end up having the exact same period, meaning the time to complete each circle is the same, and so Statement v is correct.

6A

Here the circular motion of a turntable is not at a constant angular speed, meaning there's angular acceleration, which means you can use the kinematic equations. You can use the exact same equations we use for 1-dimensioanl motion with constant acceleration, just substitute the standard variables for their rotational / angular counterparts.

According to the question, the initial angular speed is 45 rev/min, the final angular speed is zero (it comes to rest), and the time is 20 seconds (or 1/3 minute). The question wants the number of rotations, so that's the same as solving for the angular displacement. We can use the units of rev/min instead of converting to rad/s, but then the time must be converted to minutes as well, hence 1/3 minute. By keeping the units in rev/min, the displacement will already give us the number of revolutions (or rotations) instead of the displacement in radians.

To solve for displacement we can solve for acceleration first, which is equal to change in velocity divided by change in time = a = [45 rev/min – 0] / [1/3 min] = 135 rev/min^2.

Now we can use $(V_f)^2 = (V_i)^2 + 2a\Delta x$ to solve for displacement which when we rearrange to solve should become 7.5 revolutions.

Since the question asked for the number of "complete" revolutions, we just say 7.

Note: If the question had asked for the angular displacement in units of radians, we would need to multiply the 7.5 revolutions by 2π radians / revolution, giving us 15 π radians.