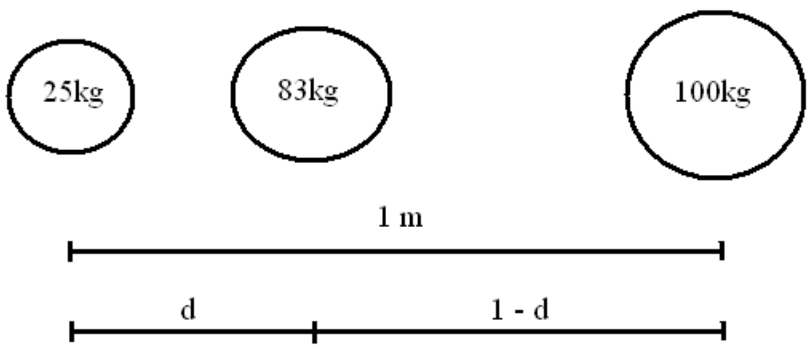
Solutions – Practice Test - PHYS 211 Final Exam (New Material)

1C

The question talks about gravitational forces, and so we need to use our new equation for gravitational force: $F = Gm_1m_2 / r^2$

What we need to realize is that the gravitational forces acting on the 83 kg object must cancel each other out, so the force pulling it towards the 25 kg mass must equal the force pulling it towards the 100 kg mass.



Using the above diagram we can create an equation representing the equal gravitational force:

$$(G)(25)(83) / d^2 = G(100)(83) / (1-d)^2$$

Notice that after simplifying the above equation not only do the G's cancel out but also the 83kg mass.

Finally we solve for d to get:

$$d = 0.33 \text{ m} = 33 \text{ cm}$$

2D

We can calculate acceleration using the equation F = ma, where F is the force of gravity, and m is the mass of any object on the planet's surface. We then expand the equation to get:

$$F = ma$$

$(GMm) / r^2 = ma$, where M is the mass of the planet and r is its radius.

Note that the mass of the object can cancel out and we are left with an equation for acceleration due to gravity:

$$a = GM / r^2$$

If we let the above equation represent the acceleration due to gravity on earth, then for the M-class planet the acceleration would be:

$a = G(2M) / (2r)^2$, which simplifies to:

$$a = GM / 2r^2 = \tfrac{1}{2}(GM / r^2)$$

Looking back at the equation for acceleration on earth we see that without even plugging in any values the acceleration on the M-class planet would be half the acceleration on earth, so we pick g/2.

3C

Remember that the escape speed is given by the equation:

$$v = \sqrt{\frac{2GM}{R}}$$

Given that the planet has double the mass and double the radius, the equation becomes:

$$v = \sqrt{\frac{2G(2M)}{(2R)}}$$

Since the factor of 2 cancels out, we end up with the same expression for escape speed.

4C

This is a straight-up question dealing with conservation of angular momentum. We need to show how initial momentum equals final momentum and then solve for the final angular velocity:

$L_i = L_f$

$I_i\omega_i = I_f\omega_f$

$(0.02)(3) = (0.02 + mr^2)\ \omega_f$

where mr^2 represents the added inertia caused by the bird and equals $(1)\ (0.1)^2 = 0.01$

Plugging that in allows us to solve for final angular velocity, which is 2.0 rad/s

5D

This problem requires the use of conservation of energy, where we use the new version of potential energy since the problem does not want us to take into account earth's gravity, or any other forces except for their own gravitational attraction. So we can write:

$U_i = U_f + K_f$ where we don't include K_i since the particle started at rest.

This then expands to become:

$(-GMm / 3) = (-GMm / 2) + (\frac{1}{2}mv^2)$

Notice that the distance is initially 3 m since we need the distance between the centers of mass, which must include the planets radius.

After the m's cancel out we can simplify and solve for v to get:

$$a\sqrt{\frac{GM}{3}}$$

6B

The question is referring specifically to comparing rotational kinetic energies, since the girl is spinning. In order to compare kinetic energies we must first have not only the different angular speeds, but also the different inertias. To figure that out, we will first have to use conservation of angular momentum $I_i\omega_i = I_f\omega_f$, which breaks down to:

$I_i\omega_i = I_f\omega_f$

$I_i\omega_i = I_f(2\omega_i)$ and after the ω_i's cancel out we have

$I_i = 2I_f$

Now we plug in these variables into the ratio of final kinetic energy to initial kinetic energy:

$$\frac{K_f}{K_i} = \frac{\frac{1}{2} I_f \omega^{2}}{\frac{1}{2} I_i \omega_i^{2}} = \frac{I_f (2\omega_i)^2}{(2I_f)\omega_i^{2}} = \frac{4}{2}$$

Thus the ratio is 2:1

7C

For this we imagine a number line with the girl at the origin (since the question asks for distance relative to her) at the boy at a distance of 10 m.

To calculate center of mass: $\frac{(40\times 0)+(60\times 10)}{100} = 6$

So we conclude that the center of mass is 6 m from the origin, or 6 m from the girl.

8C

Power, P is Work/Time. In this question the dog does work by lifting his own weight up the height of the stairs.

So we get: $P = \frac{W}{t} = \frac{mg(15\times 0.25)}{11} = 83.5W$

9B

When each child throws the ball he/she gives the ball momentum, $p = (0.35)(4.5) = 1.575$ kg m/s

Since momentum must be conserved, the child also gets 1.575 kg m/s of momentum, just in the other direction.

Furthermore, after catching the ball, the child picks up another 1.575 kg m/s of momentum.

This means in total each child gets 3.15 kg m/s of momentum.

From this we can calculate the velocity, $v = \Delta p/m = 0.063$ m/s, where the mass is 490/9.8

10D

The formula for calculating inertia for a hollow sphere (or shell) is $1/3mr^2$, however it technically doesn't matter since both objects are the same type of shape. In fact all we have to realize is that no matter what shape we're dealing with: **inertia is proportional to the square of the radius**, which means that if the radius increases by a certain factor we must square that factor to find out how much the inertia increases by. Thus, in this case, tripling the radius would increase the inertia by a factor of $(3)^2 = 9$ times the original inertia. So we pick 9I.

11A

The rule for any question like this is that the object with the larger inertia will have the smaller velocity, (angular or linear). Thus, the hollow cylinder will have the smaller linear velocity.

12D

To get the acceleration of the object we should look at the net force acting on it and set it equal to mass x acceleration….so we get:

$mg - T = ma$, where T is the tension in the chord.

The only unknown variable (other than acc.) is T, so we need anther equation that has T.

This is where we have to consider the torque caused by the cord that makes the cylindrical reel spin on its axis, using the equation: $\tau = I\alpha$, which we then have to brake down into its constituent variables:

$\tau = I\alpha$

$Tr = (\frac{1}{2}Mr^2)(a/r)$,

Where: T is tension, r is the radius, ($\frac{1}{2}Mr^2$) is the inertia of the reel, and *a* is the t angential acceleration which is the very same acceleration we are looking for.

Note also that we replace α using the equation $\alpha = a/r$

This equation can then be simplified to:

$T = \frac{1}{2}Ma$

which we then can plug back into the first equation we used:

$mg - T = ma$

$mg - \frac{1}{2}Ma = ma$

$a = mg / (m + \frac{1}{2}M) = 3.3\ m/s^2$

13A

To find the minimum mass we have to look at the forces involved that would just cause the left rope to break, which is to analyze the situation just as rotational equilibrium is about to be broken.

If we let the intersection between the right rope and the pole be our pivot point (or axis of rotation), we can calculate that where the bowling ball is located causes a counter-clockwise torque about that point (axis) given by:

$\tau = (mg)(4L/5)$,

where mg is the weight of the ball, and 4L/5 is its distance from the axis.

We also know that just as equilibrium is about to be broken and the left rope snaps, the tension in the left rope had to have been at its maximum, which would have created a clockwise torque about the pivot point, given by:

$\tau = (T_M)(L)$

Since at the breaking point of equilibrium the net torque is supposed to equal zero, the two opposing torques should equal each other:

$(mg)(4L/5) = (T_M)(L)$, and after the L's cancel out we can solve for mass

$m = 5T_M/4g$

14B

The variables needed to solve for the rotational kinetic energy of the cylinder are its inertia (which is given) and its angular speed. If, at the instant we are interested in, the bucket is moving at 8 m/s, then the tangential speed of the cylinder must also be 8 m/s, since they are connected by the rope.

Knowing this we can quickly convert from velocity to angular speed, $\omega = v/r = 200$ rad/s.

Now we simply have to plug everything in to the equation for kinetic energy:

$\frac{1}{2}I\omega^2 = 2500$ J

15A

The net force acting on the spring on the block is given by $F = -kx$. Since net force is always equal to mass x acc as well, we can set them equal to each other:

$-kx = ma$

All we have to do now is solve for a:

$a = -kx / m = 1.4$ m/s^2

16B

Remember that a shortcut for max speed comes from $v_{max} = \omega x_{max}$

If we then use the equation: $\omega = \sqrt{\frac{k}{m}}$, we can solve for max speed and get 2.6 m/s.

17B

We can calculate maximum displacement if we can get max acc, using $a_{max} = \omega^2 x_{max}$ to get max acc though we need to first understand that the force responsible for keeping the smaller block from slipping is static friction, and the largest amount of static friction we can get is given by:

$f_s = \mu n = \mu mg$.

Since friction is the only force acting on the smaller block along the horizontal axis it is also the net force and so we can set it equal to mass x acc:

$\mu mg = ma_{max}$, which simplifies to

$\mu g = a_{max}$,

where the acceleration is the maximum since we are using the maximum friction available

Then we plug in μg for the max acc into the very top equation to get:

$a_{max} = \omega^2 x_{max}$

$\mu g = \omega^2 x_{max}$,

We then replace ω using

$$\omega = \sqrt{\frac{k}{m}},$$

where m is the mass of both blocks since both are being pulled by the spring, and finally we can solve for max displacement to get 11.8 cm.

18D

Remember that for a spring (or spring-like device) the total energy stored is given by $E = \frac{1}{2}kx_M^2$, where x_M is the maximum displacement, or amplitude. All we have to do to solve then, is plug in the vibrational energy given for E, as well as the spring constant given, k, and we can solve for max displacement, $x_M = 7.9 \times 10^{-12}$ m.

19C

Remember that the total energy of a spring system can be given by the equation:

$$E = \tfrac{1}{2}k{x_m}^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

If we solve for the amplitude, x_M, this becomes:

$$x_m = \sqrt{\frac{mv^2 + kx^2}{k}} = 2.50m$$

20A

The general equation for the position function when an object is oscillating under simple harmonic motion is: $x(t) = x_M \cos(\omega t)$.

In the function actually given, we can therefore assume the angular frequency, ω, is equal to 3rad/s, (and the amplitude, x_M is equal to 5 m).

We can then determine period, T, using the relationship between ω and T, given by:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{3}s$$

21E

We can use the position function to determine the acceleration function by taking the second derivative, as shown by the following progression:

$$x = x_m \cos(\omega t)$$

$$v = -\omega x_m \sin(\omega t)$$

$$a = -\omega^2 x_m \cos(\omega t)$$

Thus we see that the magnitude of the acceleration is equal to $\omega^2 x_M$, where ω is 3 rad/s and x_M is 5 m, and we can then solve to get:

$$a_M = \omega^2 x_m = (3)^2 (5) = 45\ m/s^2$$

22C

Remember that maximum speed of any wave undergoing SHM is given by $v_M = \omega x_M$.

The distance of 3 m given in the problem from crest to trough, is the distance from the wave's highest point to its lowest point, so its amplitude will be half that distance,

and $x_M = 1.5$ m.

The frequency, f, of the waves is given when they tell us "30 full waves each minute" but we must convert to the appropriate units:

$$f = \frac{30 \text{ cycles}}{\text{min}} \times \frac{1 \text{min}}{60s} = \frac{0.5 \text{ cycles}}{s} = 0.5Hz$$

We can convert this to angular frequency, ω, using:

$$\omega = 2\pi f = 2\pi(0.5) = \pi \ rad / s$$

Lastly, we solve for v_M:

$$v_M = \omega x_M = (\pi)(1.5) = 1.5\pi \text{ m/s}$$