Solutions – PHYS 251 Exam 1 - Practice Test

1C
Although both answer choices B and C will result in an enlarged image, answer choice B provides a virtual image, which means it couldn't actually be projected onto a screen. Thus only option C could provide the desired outcome.

2A
A difficult question, to say the least, we must first recognize that we are expected to know the index of refraction in air (which is equal to 1) and the index of refraction in water (which is equal to 1.33). Using this we can find the angle of refraction in water using Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$(1)\sin(30) = 1.33\sin\theta_2$$

$$\theta_2 = 22.08^\circ$$

Note that based on the diagram given, the 60° angle is not the angle of incidence, but rather is used to determine that $\theta_1 = 30°$ is the angle relative to the perpendicular line of the water's surface.
The next step is to recognize that the light will travel horizontally both in air and in water in order to reach the 25 cm mark on the ruler, which allows us to relate the displacement along the x-axis with the following function:

$$x_{air} + x_{water} = 25$$

We can relate the vertical distance the light travels in terms of the height of the pool, h, and the depth of the water, d, which is the answer we're looking for. The vertical distance travelled in air is h-d, and the vertical distance travelled in water is just d. This allows us to set up triangles both in the air and in the water (after the light ray bends), where we now have terms for the distance along the x- and y-axis as well as the angles.

They are as follows:

<u>In air:</u>

$$\tan\theta_1 = \frac{h-d}{x_{air}},$$

$$x_{air} = (h-d)\tan\theta_1$$

<u>In water:</u>

$$\tan\theta_2 = \frac{d}{x_{water}},$$

$$x_{water} = (d)\tan\theta_2$$

This becomes:

$$x_{air} + x_{water} = (h-d)\tan\theta_1 + (d)\tan\theta_2 = 25$$

Since the only unknown variable now is the depth, d, we can solve to get d = 23cm.

3E
Using the equation for image formed by refraction:

$$s' = \frac{n_2}{n_1} s$$

where n_2 is for always the smaller value (air in this case),
where n_1 is for water,
s' is the image distance perceived (equal to 2 m in this case)
s is the actual distance we're solving for.

This rearranges and we solve for s = 2.66 m

4C
To correct near-sightedness you would want the corrected far point (or image distance) to be infinity. Using the Lensmaker's equation, and allowing the image distance to be infinity (which simply causes that term to fall to zero), we solve for the virtual image distance and get 20 cm in front of the glasses. Since these lenses/glasses were 2 cm from the eye, the total distance without glasses for the person's far point would be 22 cm.

5A
Statements II and III correctly show the relationship between distance and magnification for concave mirrors.

6A
The most important aspect of this question is that we recognize the lens must be in between the slide and the image on the screen, which means:

s + s' = 3.90 m.

Given that we have the magnification, we can write that s'/s = 38, or better yet:

s' = 38s.

By combining the above two equations we can solve for s and s', which give us:

s= 10 cm and s' = 380 cm

Lastly, we solve for f using the Lensmaker's equation and get f = 9.74 cm

7A
The safest way to solve this is to sketch ray diagrams, so long as you are sure of how the diagrams are drawn. See the review packet for examples of each scenario.

Alternatively, you could calculate image distance, s', by using the Lensmaker's equation, and by choosing an arbitrary value for the focal length, like f = 10 cm, and then determining the relative object distance, s, in each scenario. Using f = 10 cm, scenario I would have s = 15 cm, scenario II would have s = 30 cm, and scenario III would have s = 5cm. You would have to plug in the f and s values for each scenario to determine s' and then rank them from greatest to least.

8D
If the angle of refraction is smaller, it means the second medium has a larger n value than the first medium (as is the case with scenario III). Thus II must have the largest index of refraction. In option II, the situation is reversed, so the second medium must have a smaller n value than the first. In option I, the light ray is reflected rather than refracted, meaning total internal reflection, which means the index of refraction for the second medium is too low to allow refraction. This means option I must have the smallest index of refraction for the second medium. Thus we know the order is III > II > I.

9A
By drawing complimentary angles to the normal we can use geometry to determine the angle must be 10°.

However, an easier way of solving is to realize the effect of tilting the watch/mirror: Realize that if the watch were not tilted at all, meaning that the angle is 0° from the horizontal, then the reflected angle would bounce back at 30° above the horizontal instead of 10°. Also, if the watch were tilted at 30°, the light pass straight ahead at an angle of 30° below the horizontal. This means that by tilting the watch by 30°, we cause the angle of reflection to change by 60°. So we can conclude that every 1° we tilt the watch, results in a 2° change in the angle of reflection. Since the diagram shows the angle of reflection is now at 10° above the horizontal, it has changed by 20° compared to what would happen if the watch were horizontal, which must have been caused by a 10° tilt.

10A
Using Snell's Law we get:

$$n_{glass} \sin\theta_1 = n_{liquid} \sin\theta_2$$

$$(1.414)\sin(45) = n_{liquid} \sin(30)$$

$$n_{liquid} = 2.00$$

11A

We use Snell's Law twice to determine the angle of refraction for the two ends of the wavelength range, 400 nm and 700 nm, which correspond to indices of 1.55 and 1.45 respectively.

The resulting values are as follows:

$$n_{400}\sin\theta_{400} = n_{air}\sin\theta_{air}$$
$$(1.55)\sin\theta_{400} = (1)\sin(30)$$
$$\theta_{400} = 18.8°$$

$$n_{700}\sin\theta_{700} = n_{air}\sin\theta_{air}$$
$$(1.45)\sin\theta_{700} = (1)\sin(30)$$
$$\theta_{700} = 20.2°$$

As the beam travels for one meter after exiting the surface it will widen as the two extreme wavelengths are moving at different angles from one another. If we set up right triangles for each angle, using the acute angle in each case adjacent to the 1 m side length, we can find the opposite length using:

$$\tan\theta = \frac{w}{l},$$
$$w_1 = l\tan\theta_1 \text{ and } w_2 = l\tan\theta_2$$
$$w_1 = 1\tan(18.8) \text{ and } w_2 = 1\tan(20.2)$$
$$w_1 = 0.340 \text{ and } w_2 = 0.368$$

These difference between these two lengths is the width: 0.368 – 0.340 = 0.028 m.

12C

Using the equation for total internal reflection and the required critical angle of 75° from the normal line, we get:

$$\sin\theta_c = \frac{n_2}{n_1}, \text{ where } n_1 > n_2$$
$$n_1 = \frac{n_2}{\sin\theta_c}$$
$$n_1 = \frac{1.43}{\sin 75} = 1.48$$

13B
The Lensmaker's equation gets us the answer in one step. The only issue might be using consistent units.
Let f = 50 mm, s = 10,000 mm, and solve for image distance, s'.

14D
The rays shown in diagrams 3 and 4 are each false, and would in fact be appropriate in each other's diagram. Thus the ray in diagram 3 belongs on a diverging lens, and the ray in diagram 4 belongs on a converging lens.

15E
The magnification can be used to find the image distance:
s' = -1.667 x 4cm = -6.668cm
Now we use the Lensmaker's equation to solve for focal point, f = 10 cm.

16D
Let us call the two mirrors "top" and "bottom".

The object reflected in the top mirror creates image #2, which in turn, reflects via the extension of the bottom mirror to make image #3.

The object reflected in the bottom mirror creates image #5, which in turn, reflect via the extension of the top mirror to make another image (not shown) slightly beneath the location given by #4.

None of the reflections are capable of making images at positions #4 and #1.