**PHYS 250 Final Exam (New Material) - Practice Test - Solutions**

1A

The rule for any question like this is that the object with the larger inertia will have the smaller velocity, (angular or linear). Thus, the hollow cylinder will have the smaller linear velocity.

2B

The variables needed to solve for the rotational kinetic energy of the cylinder are its inertia (which is given) and its angular speed. If, at the instant we are interested in, the bucket is moving at 8 m/s, then the tangential speed of the cylinder must also be 8 m/s, since they are connected by the rope.

Knowing this we can quickly convert from velocity to angular speed, $\omega = v/r = 200$ rad/s. Now we simply have to plug everything in to the equation for kinetic energy:

$\frac{1}{2}I\omega^2 = 2500$ J

3B

The question is referring specifically to comparing rotational kinetic energies, since the girl is spinning. In order to compare kinetic energies we must first have not only the different angular speeds, but also the different inertias. To figure that out, we will first have to use conservation of angular momentum $I_i\omega_i = I_f\omega_f$, which breaks down to:

$I_i\omega_i = I_f\omega_f$

$I_i\omega_i = I_f(2\omega_i)$ and after the $\omega_i$'s cancel out we have

$I_i = 2I_f$

Now we plug in these variables into the ratio of final kinetic energy to initial kinetic energy:

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}I_f\omega^2}{\frac{1}{2}I_i\omega_i^2} = \frac{I_f(2\omega_i)^2}{(2I_f)\omega_i^2} = \frac{4}{2}$$

Thus the ratio is 2:1

4A

The net force acting on the spring on the block is given by $F = -kx$. Since net force is always equal to mass x acc as well, we can set them equal to each other:

$-kx = ma$

All we have to do now is solve for a:

$a = -kx / m = 1.4$ m/s$^2$

5B

Remember that a shortcut for max speed comes from $v_{max} = \omega x_{max}$

If we then use the equation: $\omega = \sqrt{\frac{k}{m}}$, we can solve for max speed and get 2.6 m/s.

6B

We can calculate maximum displacement if we can get max acc, using $a_{max} = \omega^2 x_{max}$ to get max acc though we need to first understand that the force responsible for keeping the smaller block from slipping is static friction, and the largest amount of static friction we can get is given by:

$f_s = \mu n = \mu mg$.

Since friction is the only force acting on the smaller block along the horizontal axis it is also the net force and so we can set it equal to mass x acc:

$\mu mg = ma_{max}$, which simplifies to

$\mu g = a_{max}$, where the acceleration is the maximum since we are using the maximum friction available

Then we plug in $\mu g$ for the max acc into the very top equation to get:

$a_{max} = \omega^2 x_{max}$

$\mu g = \omega^2 x_{max}$,

We then replace $\omega$ using $\omega = \sqrt{\frac{k}{m}}$, where m is the mass of both blocks since both are being pulled by the spring, and finally we can solve for max displacement to get 11.8 cm.

7D

Remember that for a spring (or spring-like device) the total energy stored is given by $E = \frac{1}{2} k x_M^2$, where $x_M$ is the maximum displacement, or amplitude. All we have to do to solve then, is plug in the vibrational energy given for E, as well as the spring constant given, k, and we can solve for max displacement, $x_M = 7.9 \times 10^{-12}$ m.

8C
This is a straight-up question dealing with conservation of angular momentum. We need to show how initial momentum equals final momentum and then solve for the final angular velocity:

$L_i = L_f$
$I_i\omega_i = I_f\omega_f$
$(0.02)(3) = (0.02 + mr^2)\ \omega_f$

(where $mr^2$ represents the added inertia caused by the bird and equals $(1)\ (0.1)^2 = 0.01$)

Plugging that in allows us to solve for final angular velocity, which is 2.0 rad/s

9A
The wavelength of any harmonic frequency is independent of linear density, and is based on velocity, and length. Since both wires have the same length, they will have the same wavelength.

10C
Remember that the total energy of a spring system can be given by the equation:

$$E = \tfrac{1}{2}k{x_m}^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

If we solve for the amplitude, $x_M$, this becomes:

$$x_m = \sqrt{\frac{mv^2 + kx^2}{k}} = 2.50m$$

11A
The general equation for the position function when an object is oscillating under simple harmonic motion is: $x(t) = x_M \cos(\omega t)$.
In the function actually given, we can therefore assume the angular frequency, $\omega$, is equal to 3rad/s, (and the amplitude, $x_M$ is equal to 5 m).
We can then determine period, T, using the relationship between $\omega$ and T, given by:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{3}s$$

12E

We can use the position function to derive the acceleration function by the following standardized progression for all simple harmonic motion:

$$x = x_m \cos(\omega t)$$
$$v = \omega x_m \sin(\omega t)$$
$$a = \omega^2 x_m \cos(\omega t)$$

Thus we see that the magnitude of the acceleration is equal to $\omega^2 x_M$, where $\omega$ is 3 rad/s and $x_M$ is 5 m, and we can then solve to get:

$$a_M = \omega^2 x_m = (3)^2(5) = 45\ m/s^2$$

13C

Remember that maximum speed of any wave undergoing SHM is given by $v_M = \omega\ x_M$. The distance of 3 m given in the problem from crest to trough, is the distance from the wave's highest point to its lowest point, so its amplitude will be half that distance, and $x_M$ = 1.5 m.

The frequency, f, of the waves is given when they tell us "30 full waves each minute" but we must convert to the appropriate units:

$$f = \frac{30 \text{ cycles}}{\min} \times \frac{1 \min}{60s} = \frac{0.5 \text{ cycles}}{s} = 0.5Hz$$

We can convert this to angular frequency, $\omega$, using:

$$\omega = 2\pi f = 2\pi(0.5) = \pi\ rad/s$$

Lastly, we solve for $v_M$:

$$v_M = \omega x_M = (\pi)(1.5) = 1.5\pi \text{ m/s}$$

14D

The formula for calculating inertia for a hollow sphere (or shell) is $1/3mr^2$, however it technically doesn't matter since both objects are the same type of shape. In fact all we have to realize is that no matter what shape we're dealing with: **inertia is proportional to the square of the radius**, which means that if the radius increases by a certain factor we must square that factor to find out how much the inertia increases by. Thus, in this case, tripling the radius would increase the inertia by a factor of $(3)^2 = 9$ times the original inertia. So we pick 9I.

15D

To get the acceleration of the object we should look at the net force acting on it and set it equal to mass x acceleration….so we get:

$mg - T = ma$,
where T is the tension in the chord.

The only unknown variable (other than acc.) is T, so we need another equation that has T. This is where we have to consider the torque caused by the cord that makes the cylindrical reel spin on its axis, using the equation: $\tau = I\alpha$, which we then have to brake down into its constituent variables:

$$\tau = I\alpha$$
$$Tr = (\tfrac{1}{2}Mr^2)(a/r),$$

Where: T is tension, r is the radius, ($\tfrac{1}{2}Mr^2$) is the inertia of the reel, and $a$ is the tangential acceleration which is the very same acceleration we are looking for.

Note also that we replace $\alpha$ using the equation $\alpha = a/r$

This equation can then be simplified to:

$$T = \tfrac{1}{2}Ma$$

which we then can plug back into the first equation we used:

$$mg - T = ma$$
$$mg - \tfrac{1}{2}Ma = ma$$
$$a = mg / (m + \tfrac{1}{2}M) = 3.3 \text{ m/s}^2$$