PHYS 212 Final Exam
Solutions - Practice Test

1E
If the ball is attracted to the rod, it must be made of a conductive material, otherwise it would not have been influenced by the nearby positive charge. The reason it is then attracted to the rod is due to "induction", where the electrons rearrange themselves to be as close to the positive charge as possible (or as far away from the negative charge if the rod was negatively charged). Thus we pick answer choice E.

2A
Remember that according to Coulombs Law the force acting on each sphere will be equal in size but acting in opposite directions. This is why they repel. Since the forces acting on each sphere have the same magnitude, and since the spheres have the same mass as well, the acceleration acting on each sphere will also be the same (this is comes from Newton's second law: F = ma). Thus if they each experience the same acceleration, they will move the same distance apart from each other, and answer choice A is the correct choice depicting that.

3E
We use Coulomb's law to determine the force on the negative charge due to the other two charges located on either side:

The force due to the +50μC charge is:

$$F_1 = \frac{k(50\mu C)(40\mu C)}{2^2} = 4.495N, \text{ in the } -\text{x direction}$$

where the distance is 2m based on the location of each charge along the x-axis.

The force due to the +30μC charge is:

$$F_2 = \frac{k(30\mu C)(40\mu C)}{2^2} = 2.697N, \text{ in the } +\text{x direction}$$

where the distance is 2m based on the location of each charge along the x-axis.

Thus the net force is $F_1 + F_2$ = (-4.495 + 2.697) = -1.798 N along the x-axis.

4D

The general equation for electric field is E = F/Q, where represents a test charge placed in the field. So if we solve this equation for force, we get:

$$F = EQ = (200)(-1.6 \times 10^{-19}) = -3.2 \times 10^{-17}$$

The charge is based on the charge of an electron (given on your test's data sheet) and since it is negative, it must travel in the opposite direction of the field, thus we choose answer choice D.

5A

To answer this, we must use Coulomb's law to determine the magnitude and direction of each force acting on q1 from the other charges:

$$F_{12} = \frac{k(20\mu C)(20\mu C)}{2^2} = 0.899N,\ \text{in the } +\text{y axis}$$

$$F_{13} = \frac{k(20\mu C)(40\mu C)}{2^2} = 1.798N,\ 30^\circ \text{ above the } -\text{x axis}$$

Note that we know the force from q3 is 30^0 above the –x axis since all the charges form an equilateral triangle 60 degrees apart.
In order to get the correct magnitude, we must first take the forces and place them in the respective x- and y-components:

F_{12} is already along the y-axis, so need not be altered.
$F_{13,\text{x axis}} = F_{13}\cos 30 = 1.56\text{N}$, $F_{13,\text{y axis}} = F_{13}\sin 30 = 0.899\text{N}$

Thus using vector analysis we see that the total force along each axis is:

y-axis: 0.899 + 0.899 = 1.798 N
x-axis: 1.56 N

The magnitude using Pythagorean theory: $F = \sqrt{(1.798)^2 + (1.56)^2} = 2.38N$

Thus we choose answer choice A.

6E
As we saw during the review, we can break down the 3Q charge so that it is distributed evenly across the three quadrants. The purpose being that we can start to use symmetry to help analyze the resultant electric field.

The left rod simply has a charge of 1Q located along the Top-Right quadrant.
If we distribute the 3Q charge evenly, it would look as though we have a charge of 1Q along the Top-Left quadrant, a charge of 1Q along the Top-Right quadrant, and a charge of 1Q along the Bottom-Right quadrant.

Although we don't have an exact value for the electric field produced due to each quadrant, we can assume that the fields from the Top-Left and Bottom-Right quadrants will cancel each other out (since electric fields are vectors) and thus we are left with just a 1Q charge in the Top-Right quadrant, which is exactly what we have in the original left rod, and thus their fields are the same strength.

7E
Since these are insulating sheets (not conducting) the charge will be emitted from each side of the sheet and travel evenly in each direction. Thus the field in regions II and III will be non-existent since the vectors from each sheet will cancel out in those regions. Only in regions I and IV, will the electric field not cancel out and therefore be strongest.

8A
Where the density of the field lines has been halved, we can assume the electric field is half its original value. Since the force is proportional to the field strength, when the field is halved, so too is the force, thus it becomes 5 pN. Also, since we have switched from a proton to an electron, the charge has flipped and so too must the direction of the force, thus we move to the right instead of the left.

9C
According to the packet, for a solid insulating cylinder there are two equations we use for determining electric field based on whether we are inside or outside the cylinder. According to the equations, we find that inside the cylinder the field increases linearly, and outside the cylinder it decreases asymptotically. Thus only answer choice C follows this pattern.

10A
This type of problem needs a little visualization before trying to solve. <u>Any electric field moving through the xy plane is actually headed along the z-axis</u>. Once we can see that, we can calculate the specific flux using the equation $\Phi = EA = (16)(2) = 32$. Note that we use the z-component for the electric field, 16k N/C.

11E
The most effective use of Gauss' Law is that the total flux through an enclosed surface is given by total internal charge, q, divided by the constant ε_0. Since we are not given the total charge, we can substitute for it using $q = \lambda L$, where the charge density, λ, is q/L. Thus we can pick answer choice E to represent the total flux.

12C
For any conducting spherical shell, the inside surface will have the same charge as the point charge inside the sphere but will have the opposite sign. Thus the inner surface must be -5μC. However, since the +5μC inside the sphere is not located at the center, the charge will not be uniformly distributed along the inner surface. Another rule for conducting spherical shells, is that the sum of the inner and outer surface charges must equal the excess charge of the sphere.
So we get:

Inner + Outer = Excess

-5μC + Outer = +3μC

Outer = +8μC

The outer surface charge will be uniformly distributed, unlike the inner surface charge, and thus we pick answer choice C.

13B
Outside of the cylinder, the field decreases inversely proportional to the distance form the center. Thus if the distance doubles, the field will be halved.

14A
The uniformly distributed charge can be thought of as a point charge located at the center of the sphere. According the equation for electric potential of a point charge, the potential decreases as the distance form the point increases. Thus the closer to the center you are, the greater the potential will be, and so we pick answer choice A.

15E

The only equation relating potential energy and electric potential is $\Delta V = \Delta U/q$. This means to calculate the change in potential energy, we need to multiply ΔV with q. Note that in this particular problem, q represents the negative charge that is moved from the x-axis to the y-axis.

The change in electric potential, ΔV, is a scalar quantity that is essentially only interested in the fixed point charge and its distance, r, where we measure the potential. In this particular problem, the initial and final locations of the moving charge are the same distance from the fixed charge, thus the potential at each location is identical. This means that the change in potential as the negative charge moves from its initial to its final location is zero, and thus the change in potential energy is also zero.

16D

This scenario is indicative of a positive point charge, which decreases in field strength the further you are from it. The potential of such a point charge will also decrease as you move away from the charge, and thus answer D is the appropriate choice.

17B

The answer choice differ in the way they express the distance form the point Y, to any place on the rod, given by the position, x. Essentially, we can think of the dashed line as the distance we are trying to express, which we can calculate using Pythagorean theory: where the distance along the x-axis is given by L-x, the distance along the y-axis is given by y, and the vector distance will then be: $\sqrt{(L-x)^2+(y)^2}$

Thus we choose answer choice B as it is the only option that expresses the distance from the rod to point Y in this correct format.

18B

The energy used to pull apart the plates is transferred into stored energy in the capacitors, so we expect the energy stored to increase. Based on our equations for capacitors, the distance between plates is inversely proportional to the capacitance, and so doubling the distance halves the capacitance, which doubles the energy stored within them. Thus we choose answer choice B.

19E

The key to such problems is to find the equivalent capacitance for each row and then to combine the rows and find their equivalent capacitance.

The top row has 3 identical capacitors in series, and so we can solve for their equivalent capacitance using:

$$\frac{1}{C_{eq,\,top}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C},$$

$$C_{eq,\,top} = \frac{C}{3} = 2\mu F$$

The middle row has 2 identical capacitors in series, and so we can solve for their equivalent capacitance using:

$$\frac{1}{C_{eq,\,middle}} = \frac{1}{C} + \frac{1}{C},$$

$$C_{eq,\,\mathrm{middle}} = \frac{C}{2} = 3\mu F$$

The bottom row has just 1 capacitor, and so we leave its capacitance alone,

$C_{\mathrm{bottom}} = 6\mu F$

We are now left with an equivalent circuit of just 3 capacitors, one on each row, that are in parallel. To get one final equivalent capacitance, C_{EQ}, for capacitors in parallel, we use the equation:

$$C_{EQ} = C_{Top} + C_{Middle} + C_{Bottom} = 11\mu F$$

20D
As usual with circuits such as these, we need to simplify the circuit to determine the equivalent capacitance, C_{EQ}. Capacitors 2 and 3 are parallel to each other and can be combined simply be adding the capacitance values together to get C_{23} = 15 μF. The circuit is now equivalent to two capacitors in series, capacitor 1 with capacitance 30μF, and the combined parallel capacitors with equivalent capacitance 15μF. The last step in simplifying the circuit is to determine the equivalent capacitance of the 30μF and 15μF capacitors in series, which is:

$$\frac{1}{C_{eq}} = \frac{1}{30} + \frac{1}{15},$$
$$C_{eq} = 10\mu F$$

Now that we have our equivalent capacitance of 10 μF, we can use the equation below to determine total charge for this circuit:

$$Q_{Total} = C_{EQ}V = (10\mu F)(120V) = 1200\mu C$$

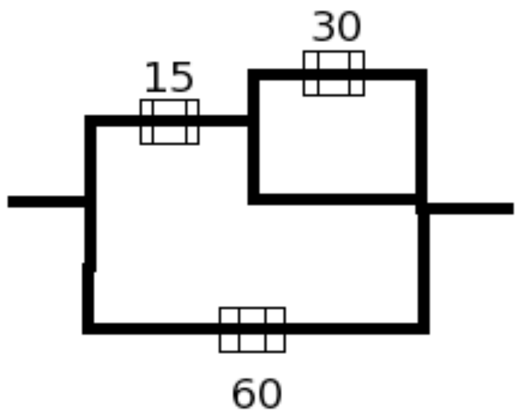
If we then return back to the simplified version of the circuit that had capacitor 1 in series with the equivalent capacitance of 15μF (from capacitors 2 and 3 in parallel), we can then see that since the capacitors are in series, the rule is that the charge going across each capacitor is the same and equal to the total charge. Thus the charge across capacitor is the same as the total charge and equal to 1200μC.

21E
In order to use the equation for discharging, we should consider the amount of charge *remaining* after three time constants, which would have to be $q(t)/q_0$. Also we can substitute time, t, with 3τ, which is three time constants. We can also substitute RC, which is also equal to τ, and we get:

$$\frac{q(t)}{q_0} = e^{\frac{-t}{RC}} = e^{\frac{-3\tau}{\tau}} = e^{-3} = 0.0498 = 4.98\%$$

This means that after three time constants, we have 4.98% left, which means that charge has been reduced by 95.02%.
Note that the fact that charge had been reduced by 63% after one time constant was useless information if you solve it this way.

22D
Using the appropriate equations for resistors in parallel, the two resistors at the top left of the circuit can be combined into equivalent resistance of 15 ohms. Also, the two resistors in series at the bottom are the equivalent of 60 ohms. Thus the circuit can be redrawn as shown velow:



At this stage the current can get from “a” to “b” by travelling through either the 15 ohm resistor or the 60 ohm resistor. It does not need to go via the 30 ohm resistor at the top right. Which means that for all intents and purposes we can ignore it. So now we really have a simple parallel circuit with resistors of 15 and 60 ohms, and an equivalent resistance of 12 ohms.

23D
Looking at the currents entering and leaving any of the junctions where the wires meet gives the following equation: $I_2 + I_3 = -I_1$.
This can be rewritten to get answer D.

24D
We treat the resistors in series on the top half of the circuit as one resistor of 12 ohms, and the resistors at the bottom as one resistor of 596 ohms. So the circuit is really a parallel circuit with equivalent resistance of 11.8 ohms.

25D
The voltage drops from 9 V to 8.5 V due to the internal resistance. This 0.5 V lost within the battery can be set equal to V = IR, allowing us to solve for the current, I, to get:

$$I = V/R = 0.5/0.1 = 5 \text{ A}$$

26A
Remember that going across a battery from + to – makes the potential drop negative, and going against the direction of current when crossing a resistor makes the drop positive. Only answer A obeys these rules.

27D
Initially, the capacitor will not affect the resistance of the circuit, and you can pretend that it is not even there. Thus the current in the circuit will be I = V/R, regardless of the capacitance, and is thus 18/2 = 9 Amps.

28D
For any RC circuit, you can use the loop law to include the capacitor (assuming it has charge) so that it looks as follows:

$$\varepsilon - Ri - \frac{q}{C} = 0$$

This can be rewritten to solve for current:

$$i = \frac{\varepsilon - \dfrac{q}{C}}{R} = 4$$

29D
Since the current density, J, is not uniform along the cross-section of the conductor, we must use the equation:

$$I = \int J \bullet dA$$

We must replace J with the function given in the question, and dA represents the derivative of the area of the cylinder's cross-section, 2πr, giving us:

$$I = \int_0^R \left(\frac{Kr^2}{2R}\right)(2\pi r)dr = \frac{K\pi}{R}\int_0^R r^3 = \frac{K\pi R^4}{4R} = \frac{K\pi R^3}{4}$$

30A
The appropriate equation we need to get resistivity based on the information given is:

$$R = \frac{\rho L}{A}$$

We already have the area, A, and the length, L, so we just need resistance, R, which we easily get from the equation R = V/I = 1/0.5 = 2.0 ohms.

Then we plug in our resistance, R, and solve to get:

$$\rho = \frac{RA}{L} = \frac{(2.0)(10^{-6})}{0.3} = 6.67 \times 10^{-6}$$

31A
The most basic definition of current is: I = C/s
Thus, the time in seconds is given by s = C/A = (5)/(10) = 0.5 s

32C
Remember that resistivity is a property of a material, and so both wires have the same resistivity (this rules out A).

The graph shows that at any given voltage the current in wire A is greater than that of wire B. This can only happen if wire A has a smaller resistance, since current and resistance are inversely proportional. That means that statement ii is correct. We should also remember that if everything else is held constant, the shorter the conductor is, the smaller the resistance will be. So since wire A has the smaller resistance, we can assume it has a shorter length, and thus statement v is also correct.

33D
Energy dissipation is the same thing as power. So let's see the equations for power:

$$P = V_0 I = I^2 R = \frac{V^2}{R}$$

We can analyze each answer choice as follows:

(a) False – if you half V (with R constant) the power decreases by a factor of one fourth.
(b) False – if you half I (with R constant) the power decreases by a factor of one fourth.
(c) False – if you half R (with V constant) the power will double.
(d) True – according to P = I^2R, if you half R (with I constant) the power will half.
(e) False – if you half both V and I the power decreases by a factor of one fourth..

34D
The Biot-Savart Law for magnetic fields from a current is used to determine the direction of the field. By pointing your thumb in the direction of the current, your fingers curl in the direction of the magnetic field. From this we determine that the two wires on the right going into the page will cancel each other out at point P (the top right wire produces a field to the left, and the bottom right wire produces an equal field to the right). The other wires both produce magnetic fields pointing right at point P.

35B
The equation for any circular loop is given by:

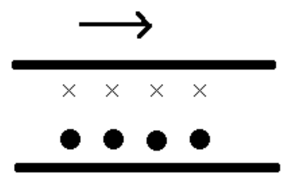
$$\vec{B} = -\frac{\mu_0 i}{2R} = 2.0 \times 10^{-5} T$$

However, since we are only dealing with half a loop, we will get half the amount of magnetic field, B/2, which equals 1.0×10^{-5} T.

36E
For a charge/current moving in a circular path, the Biot-Savart Law says to point your thumb in the direction of motion and your fingers will curl in the direction of the magnetic field. Doing this with the particles causes your fingers to curl into the page at the center of the circle. However, all rules are reversed when the charge is negative, and so the field will point out of the page instead.

37B

For each wire the direction of the magnetic field can be determined by pointing your thumb along the wire and curling your fingers in the direction of the field. Since both currents are parallel (and assuming the current moves to the right), the top wire produces a field going into the page, while the bottom wire makes a field going out of the page, as shown below:



This means that at the midway point, which is 0.2 m from each wire, the fields will oppose each other, and the net magnetic field will be the difference between the two. For a straight wire carrying current, the magnetic field at a distance, R, from the wire is:

$$\vec{B} = -\frac{\mu_0 i}{2\pi R}$$

Thus the net field will be $B_1 - B_2$:

$$\vec{B}_1 - \vec{B}_2 = \left(-\frac{\mu_0(4)}{2\pi(0.1)}\right) - \left(-\frac{\mu_0(5)}{2\pi(0.1)}\right) = 2 \times 10^{-6} T$$

38C

Remember that the capacitance is decided once the capacitor is built, and cannot be changed after that point. Thus, plate separation, plate area, and the insertion of a dielectric material, are the only factors that influence capacitance.

39B

Ampere's Law tells us that the line integral for the magnetic field within the closed loop is given by $\mu_0 I_{enc}$, which in this instance is equal to $\mu_0(5)$, as the two wires going into the page cancel out two of the three wires going out of the page, leaving only one 5A wire remaining. Since $\mu_0 = 4\pi \times 10^{-7}$, $\mu_0 I_{enc} = 20\pi \times 10^{-7} = 2\pi \times 10^{-6}$ Tm.

Lastly, remember that the sign of the line integral is determined by comparing the direction of the path with the direction of the magnetic field created by the current-carrying wire. The path is clockwise, and since the current is moving out of the page, the right-hand-rule shows the magnetic field is directed counterclockwise. Thus, since the path and the magnetic field are moving in opposite directions, the line integral needs to be negative.

40A
The RC circuit has an uncharged capacitor initially connected to a battery, thus the appropriate equation we need to use is for **charging** a capacitor:

$$q(t) = q_0(1 - e^{-\frac{t}{RC}})$$

Since we are looking to solve for time, t, when the charge, q(t) is 75% of its maximum value, this is equivalent to substituting q(t) with $0.75q_0$ to get:

$$0.75q_0 = q_0(1 - e^{-\frac{t}{RC}})$$

The maximum charge cancels on both sides, and we simplify to get:

$$-0.25 = -e^{-\frac{t}{RC}}$$

The negatives cancel on each side, and we can replace 0.25 with 1/4:

$$1/4 = e^{-\frac{t}{RC}}$$

$$\ln(1/4) = \ln(e^{-\frac{t}{RC}})$$

$$\ln(1/4) = -\frac{t}{RC}$$

Lastly we solve for time, t, and end up with:

$$t = -RC\ln(1/4)$$

Note that although this answer looks very similar to B, it is not the same, as our solution has a negative sign in front of it. Given that none of the answer choices have negative signs, we have to try and be creative, and recognize that if we take the reciprocal of the value within the natural log function it flips the sign,; meaning that ln(1/4) is equal to –ln(4).

Thus the answer ends up being:

$$t = RC\ln(4)$$

41D
Let's assume the direction of the current to be the positive direction. The induced EMF can be calculated using:

$$EMF = -L\frac{di}{dt} = -(1.5)(+3) = -4.5V$$

Thus the induced EMF is 4.5 V and is in the opposite (negative) direction of the current.

42C
The current in the wire will induce a magnetic field going into the page on the right side of the wire. This causes an induced current in the loop such that the induced current creates a magnetic field opposing the one from the wire (meaning that induced magnetic field from the loop must travel up out of the page). This would happen if the current in the wire were moving counterclockwise according to the right-hand rule.

43D
The equation we're looking for to connect these variables is:

$$\omega = \frac{1}{\sqrt{LC}} = 2\pi f$$

where f is the frequency mentioned in the problem.

All we need do is solve for L:

$$L = \frac{1}{4\pi^2 f^2 C} = 5.07 \times 10^{-7} H \approx 0.5\mu H$$

44B
The equation for any circular loop is given by:

$$\vec{B} = -\frac{\mu_0 i}{2R} = 2.0 \times 10^{-5} T$$

However, since we are only dealing with half a loop, we will get half the amount of magnetic field, B/2, which equals 1.0×10^{-5} T.

45E
Using the equation for frequency:

$$\omega = \frac{1}{\sqrt{LC}} = 2\pi f$$

we notice that frequency is proportional to the inverse square root of the capacitance, C:

$$f \propto \frac{1}{\sqrt{C}}$$

which when we solve for capacitance we get:

$$C \propto \frac{1}{f^2}$$

In the problem we're told that we double the frequency, and so according to the above proportion, doubling f causes C to become C/4.

46C
There are many ways to calculate power (which is the same thing as energy dissipation rate), but the one we need to use involves current and voltage:

$$P = iV = (7.5 \times 10^{-3})(120 \times 10^{3}) = 900 \text{ Watts}$$

47B
A bit of a trick question really. Since the circuit is in series, the current across all three devices will be the same, allowing the EMF to be in phase with the capacitor, so it's frequency will be the same.

48C

This is a standard simple RL circuit that has just been switched on. For problems like this we need to look at the current initially (t = 0) and after a long time (t >> τ).

> At t=0 the inductor tries to resist a change and so the current is basically non-existant.
>
> At t >> τ the inductor is done resisting anything, allowing current to pass through as though it weren't even there.

To find what sort of time would be considered "a long time" such that t >> τ, we first need to find our time constant, τ.

$$\tau = \frac{L}{R} = \frac{0.1}{1} = 0.1s$$

This means that after a few time constants (anything greater than say 5 x τ, or 0.5 secs) would be a long enough time.

Let's now analyze the answer choices:

(a) After 10 secs the only device with any resistance at all is the resistor and so it gets all the potential drop available.

(b) After a long enough time we can act almost as if the inductor isn't there and so there is virtually no resistance and thus no drop in potential as current crosses freely.

(c) 10 secs is definitely a long time relative to the time constant of 0.1 s, and so the current flows normally across the resistor. This means the power or energy dissipated across the resistor would be at a maximum (certainly not negligible).

(d) The current at any time can be calculated with:

$$i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{t}{\tau}}\right) = 5.67A$$

(e) This is exactly the case at the very early times relative to the time constant.

49D

If we find the resultant velocity, v, its vector is 13 m/s. This can be plugged into the equation for magnetic force: $F = qvB = 1.04 \times 10^{-17}$ N, where q is the charge of an electron given on your data sheet as 1.60×10^{-19} C.

50B

This one's just straight from the packet. You should definitely be familiar with each of Maxwell's Equations and what they specifically refer to.

51E

If there is no magnetic field inside the shell, then according to Gauss' law there will no be no field inside the shell either, and thus the current will be zero.

52A

If we say that into the page is positive, we can evaluate the changes in magnetic flux to see what direction the induced magnetic field would need to point.

A: No change in flux occurring so no induced current.

B: $\Delta B = +1 - 0 = +1$

This means the induced magnetic field must be -1 (out of the page) to oppose the existing change in flux. To make the induced magnetic field go out of the page, we curl our right hand fingers **counter-clockwise** and our thumbs point up out of the page.

C: Although there is a magnetic flux, it is constant, and thus no induced current.

D: $\Delta B = 0 - +1 = -1$

This means the induced magnetic field must be +1 (into the page) to oppose the existing change in flux. To make the induced magnetic field go into the page, we curl our right hand fingers **clockwise** and our thumbs point down into the page.

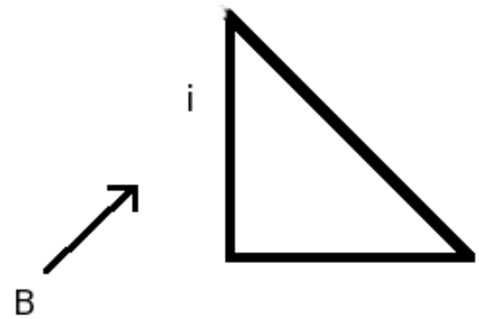
53D
The equation for induced magnetic fields is:

$$\oint B \cdot ds = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

Notice that in this scenario the induced magnetic field will have the same sign as the changing electric flux. This means that the induced electric field which results from the induced magnetic field will point in the **same** direction as an increasing electric field (which would be away from the person). Since the electric field that causes this is not increasing but decreasing instead, we know that the induced magnetic field will do the opposite, to make sure that the induced electric field actually points towards the person. The only way an induced magnetic field could induce an electric field pointing towards the person is if it moves counterclockwise. Curl your right hand in a counterclockwise direction and notice that your thumb points towards you, which is the direction the induced electric field would have to follow.

Note that in a question such as this, the magnetic field would always be perpendicular to the electric field, and so the answer could only be clockwise or counterclockwise.

54C
Based on the description given, the only way the right triangle can also be an isosceles triangle is if the other angles are both 45 degrees. Looking at the diagram below, we see that the magnetic field hits the two equal sides at a 45 degree angle.



For the vertical side, the component of the magnetic field that is perpendicular to the wire would be given by Bcos(45), and for the horizontal side it would be Bsin(45). Since the sine and cosine of 45 are equal this distinction doesn't matter, but it might matter for a different problem so it's good to be aware of it.

We can now use the equation for force caused by a magnetic field on a current carrying wire for each side:

$$\vec{F}_1 = i\vec{L} \times \vec{B} = iLB\cos(45) = 0.00247N$$
$$\vec{F}_2 = i\vec{L} \times \vec{B} = iLB\sin(45) = 0.00247N$$
$$F_{Total} = F_1 + F_2 = 0.00495N$$

55E
Inside and outside the toroid, the magnetic field is zero, so only (a) and (e) are viable answers. We also should know that magnetic field will decrease as the radius increases within the shell of the toroid (but does not reach zero), and so (e) is the only option.

56E
If the magnetic field is in the z-axis, the only way it could rotate would be perpendicular to that axis, meaning that rotation could only happen around the x- or y- axis.