# **Solutions - Practice Test - PHYS211 Final Exam (Old Material)**

1A

Although the numbers on the graphs don't exactly fit with what they should actually be, the pattern or structure of the graph is what we should be looking at.

So, based on the graph for acceleration, <u>we</u> can see that for the first 3 secs we have a constant negative acceleration (-2 m/s<sup>2</sup>). This should equate to a negative or downward slope for velocity during the first 3 secs. Only answers (a) and (b) show this.

From t = 3 to t = 6, the acc becomes positive (+1 m/s<sup>2</sup>), and so we expect a positive or upward slope, which again is present in both (a) and (b). However, since the magnitude of the accelerations is larger for the first three seconds we should expect to see a steeper slope from t = 0 to t = 3 than from t = 3 to t = 6. This only happens in graph (a).

2B

Remember people, the formula for acceleration is:

Average Acceleration, 
$$\bar{a} = \frac{\Delta v}{\Delta t}$$

However, we can't just plug in the velocities given since they're not all in the same dimension. The easiest way is to use the vector coordinates for both velocities:

$$V(initial) = (0i + 4.0j) \text{ m/s}, \quad V(final) = (3.0i + 0j)$$

$$\Delta V = V(\text{final}) - V(\text{initial}) = (3.0i - 0i) + (0j - 4.0j) = (3.0i - 4.0j) \text{ m/s}$$
  
The next step is to write that vector in terms of its magnitude, using Pythagorean Theory to get  $\Delta V = 5 \text{ m/s}$ .

Finally, we plug into the equation for acceleration:

$$a = \frac{5m/s}{2s} = 2.5 \, m/s^2$$

3E

The area under the graph, the integral, from t=0 to t=3, will give us the displacement. The area equals 7m, so the final position at t=3 is 12m, since the particle was originally at x=5m.

4D

Here the definition for average speed is total distance / total time. The information given has everything we need except the distance travelled during the first portion of the trip, so we must calculate that first:

$$\Delta x_1 = v_1 t_1 = (80 \text{km/hr})(1/2 \text{hr}) = 40 \text{km}$$

So we get average speed = [40 + 0 + 120] / [1/2 + 1/6 + 2] = 60

5C

There are two parts to this problem, the first requires figuring out the displacement and speed the cheetah reaches after the first 1.5s:

### x-axis

$$V_0 = 0 \text{ m/s}$$
  
 $V = ?$   
 $a = 15 \text{ m/s}^2$   
 $t = 1.5 \text{ s}$   
 $\Delta x = ?$ 

3 out of 5 is enough and so we can solve for both using:

$$\Delta x = v_0 t + \frac{1}{2} a t^2 = 16.875 m$$

$$v = v_0 + a t = 22.5 m/s$$

Since we need to know how long it will take to reach 60m, we must now determine how much farther the cheetah needs to travel: 60 - 16.875 = 43.125 m at 22.5 m/s. The speed is constant, so the time required is:

$$acnt = 43.125m / 22.5m/s = 1.9s$$

Thus the total time taken to reach 60m equals = 1.5s + 1.9s = 3.4s

This is essentially a question dealing with 2-dimensional motion where both dimensions (x-axis and y-axis) have their own constant acceleration.

We are concerned with finding the y-coordinate, which is like solving for  $\Delta y$ . The only problem is we are only given initial velocity and acceleration for the y-axis (2 variables are not enough). Remember, when dealing with 2-D motion, you can always use the same time found in the other axis, in this case we can use the time for motion along the x-axis. All we need to do is find out what that time is:

#### x-axis

$$V_0 = 0 \text{ m/s}$$
  
 $V =$   
 $a = 6.0 \text{ m/s}^2$   
 $t = ?$   
 $\Delta x = 27 \text{ m}$ 

3 out of 5 is enough and so we can solve for time using:

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

This gives us t = 3.0 s, which we can use for calculations involving the y-axis:

#### y-axis

$$V_0 = 4.0 \text{ m/s}$$
  
 $V =$   
 $a = 4.0 \text{ m/s}^2$   
 $t = 3.0 \text{ s}$   
 $\Delta y = ?$ 

Again we use:

$$\Delta y = v_0 t + \frac{1}{2} a t^2$$

To get  $\Delta y = 30 \text{ m}$ 

The velocity as the ball leaves the table is completely horizontal, so we should first look to the x-axis to calculate the value:

$$V_x = \Delta x/t = 1.52/t$$

Since we don't know the time, we must use the y-axis to get that information:

## <u>y-axis</u>

$$V_0 = 0 \text{ m/s}$$
  
 $V =$   
 $a = -9.8 \text{ m/s}^2$   
 $t = ?$   
 $\Delta y = -1.5 \text{m}$ 

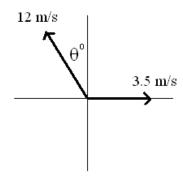
Again we use:

$$\Delta y = v_0 t + \frac{1}{2} a t^2 = \frac{1}{2} a t^2$$
$$t = \sqrt{\frac{2\Delta y}{a}} = 0.55 s$$

Now we have time, we can plug it back into the x-axis equation to get velocity:

$$V_x = \Delta x/t = 1.52/0.55 = 2.76 \text{ m/s}$$

The only way to travel due north is to counter the vector pointing east with a vector that has an x-component pointing west, as shown below:



Specifically, we must ensure that x-component of the 12 m/s vector is equal to 3.5 m/s, so that the only net vector is the y-component of the 12 m/s which points north.

So we get:

 $12\sin\theta = 3.5$ 

$$\theta = \sin^{-1}\left(\frac{3.5}{12}\right) = 17^{\circ}$$

Which gets us 17° west of north.

## 9D

A typical 2-D question where we're asked to determine  $\Delta y$  but don't have enough variables to do so. So, as usual, when dealing with projectile motion we can use the time from the other dimension, x-axis, to help us out.

## x-axis

$$V_x = \frac{\Delta x}{t},$$

SO

$$t = \frac{\Delta x}{V_x} = \frac{12}{21\cos 25^\circ} = 0.63 \, \text{ls}$$

Now we can go to the y-axis and solve for  $\Delta y$ :

## y-axis

$$V_0 = 21\sin 25^\circ = 8.87 \text{ m/s}$$
  
 $V = a = -9.8 \text{ m/s}^2$   
 $t = 0.631 \text{ s}$   
 $\Delta y = ?$ 

We then use:

$$\Delta y = v_0 t + \frac{1}{2} a t^2 = 3.65 \text{ m}$$

Remember that whenever you're asked to solve for final speed/velocity in any projectile motion problem, you must consider both the x-and y- components of the vector.

So for this particular question, we can start with either the x- or y-axis.

For the x-axis, the final velocity is the same as initial velocity since there's no acceleration along the horizontal axis.

Therefore  $V_x = V\cos\theta = 28\cos 55^\circ = 16.06 \text{ m/s}$ 

For the y-axis:

## <u>y-axis</u>

$$V_0 = 28\sin 55^\circ = 22.9 \text{ m/s}$$
  
 $V = ?$   
 $a = -9.8 \text{ m/s}^2$   
 $t = 3 \text{ s}$   
 $\Delta y =$ 

And we can use the following to solve for  $V_y$ :

$$V_y = V_0 + at = -6.46 \text{ m/s}$$

Lastly, we solve for the resultant vector of the two velocity components using Pythagorean theory to get V = 17.3 m/s.

Despite how morbid this question might be, the physics behind it is quite straightforward. The scenario suggests a 1-D motion type-problem where we're solving for time and ae given the following data:

<u>y-axis</u>

$$V_0 = +8 \text{ m/s}$$
  
 $V =$   
 $a = -9.8 \text{ m/s}^2$   
 $t = ?$   
 $\Delta y = -420 \text{ m}$ 

The only problem is that to solve for t we would have to set up a quadratic eqn; which is time consuming, unless you plot the function on your graphing calculator and see where it crosses the x-axis.

Alternatively, we can first solve for V and then solving for time will be far easier.

So we use:

$$v^2 = v_0^2 + 2a\Delta x$$

And get V = -91.08 (remember that it's negative because the man is falling down, not up)

Now we can use any equation to solve for time:

$$v = v_0 + at$$
  
$$t = \frac{v - v_0}{a} = \frac{-91.08 - 8}{-9.8} = 10.11s$$

Since the two balls have different initial speeds, but the same constant acceleration due to gravity, the easiest way to solve this problem is to create position functions for each of the balls and use them to solve for time.

$$(1)\Delta y_1 = 15t + \frac{1}{2}at^2$$

$$(2)\Delta y_2 = -10t + \frac{1}{2}at^2$$

The question asks for when the two balls will be 50m apart, so that means we want:

$$\Delta y_1 - \Delta y_2 = 50$$

Which can be re-written as:

$$(15t + \frac{1}{2}at^2) - (-10t + \frac{1}{2}at^2) = 50$$

$$25t = 50$$

$$t = 2s$$

This is a particularly difficult question to deal with especially when you consider that you are not even given the height of the building. With only the initial velocities of the two stones, this question would be impossible to solve except that in this particular case the two velocities have equal magnitude. Why is that important?.....We'll get to that in a minute. First, we must recognize a rule about projectile motion that states that an object thrown up with any initial velocity, V, will be traveling at a velocity of negative V when it falls back down to the height that it was initially thrown from. In other words, or to give an example relative to this question, the stone that is thrown upwards at 15 m/s will reach its max height and then start falling back down towards earth with increasing negative velocity...and when it gets to the position from which it was released (the start point of this question) it will be traveling at -15 m/s.

So why is this information helpful?...Well it tells us that the stone thrown upwards will eventually return to its starting point after some amount of time, and that at that exact time it will the same velocity that the other stone had when it was initially released (-15 m/s)....which means that it will follow the exact same path as that stone and will take the exact same amount of time to reach the ground as that stone did when it was initially released.

And so to find out how much time will pass between the two stones hitting the ground, we only have to calculate how much time the stone thrown upwards spends in the air before returning to its initial release point. For this scenario we have the following:

### <u>y-axis</u>

$$V_0 = 15 \text{ m/s}$$
  
 $V = -15$   
 $a = -9.8 \text{ m/s}^2$   
 $t = ?$   
 $\Delta y = 0$ 

Yes..we even have 4 variables as a reward for understanding physics.

And we can use any equation to solve for time:

$$v = v_0 + at$$
  
$$t = \frac{v - v_0}{a} = \frac{-15 - 15}{-9.8} = 3.06s$$

14D

This question asks for the speed and represents 2-D motion. However, the equations for position in both axis are given, so we cannot rely on the typical equations for motion that we have used up to this point. The easiest solution is take the derivative of each position function to get velocity for each axis and then to calculate the specific velocities at t = 2s:

$$\frac{d}{dt}x(t) = v_x(t) = 4, \quad \frac{d}{dt}y(t) = v_y(t) = 6t - 9,$$
 $v_x(2) = 4m/s \qquad v_y(2) = 3m/s$ 

Thus the resultant vector velocity (speed) is:  $\sqrt{4^2 + 3^2} = 5m/s$ 

15B

The y-axis is the best approach to analyzing the variables since it is due to the acceleration of gravity along the y-axis that the ball drops back down:

So we get:

y-axis

$$V_0 = v_0 sin\theta$$

$$V =$$

$$a = -g$$

$$t = ?$$

$$\Delta v = 0$$

We then use the below equation which we can simplify to solve for time:

$$\Delta y = v_0 t + \frac{1}{2}at^2$$

$$0 = (v_0 \sin \theta)t + \frac{1}{2}(-g)t^2$$

$$\frac{1}{2}gt^2 = (v_0 \sin \theta)t$$

$$\frac{gt}{2} = v_0 \sin \theta$$

$$t = 2v_0 \sin \theta/g$$

The easiest approach will be to create position functions for each ball. First though, we must consider that the balls collide at some point between 0 and 50 m, called y.

That means that for ball A its displacement will be  $\Delta y_A = y - 0 = y$ And then for ball B its displacement will be  $\Delta y_B = y - 50$ 

Now that we've clarified how the two displacements relate to the point of collision, y, we can go ahead and write our position functions:

(1)
$$\Delta y_A = y = V_0 t + \frac{1}{2} a t^2$$
  
(2) $\Delta y_B = y - 50 = 0 t + \frac{1}{2} a t^2$ 

If we then solve eqn (2) in terms of y, and set it equal to y in eqn (1) we get:

$$y-50 = 0t + \frac{1}{2}at^{2}$$

$$y = \frac{1}{2}at^{2} + 50$$

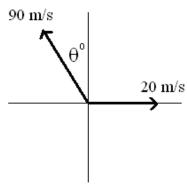
$$y = \frac{1}{2}at^{2} + 50 = V_{0}t + \frac{1}{2}at^{2}$$

$$50 = V_{0}t$$

Thus when we plug in t = 2.5 s we get:

$$V_0 = \frac{50}{2.5} = 20 \, m/s$$

The appropriate vector diagram should look as follows:



To counter the easterly wind, the plane's velocity vector must have an x-component that is equal to 20 m/s. So we get:

$$90\sin\theta = 20$$

$$\theta = \sin^{-1}\left(\frac{20}{90}\right) = 12.84^{\circ}$$

Now that we have the angle we can calculate the velocity component that heads due north using  $V=90cos12.84^\circ=87.7~m/s$ 

Finally we use the equation for average velocity:

$$V = \frac{\Delta x}{t},$$

$$t = \frac{\Delta x}{V} = \frac{350000}{87.7} = 3988.6s = 66.5 \,\text{min}$$

## 18A

We can split this problem up into two parts, each one dealing with a separate axis.

For both the x- and y- axes, we have two forces (one given, one unknown) and we get the following equations:

$$F_x = -2.5 + x = ma_x$$

$$F_x = -2.5 + x = (1.7)(3.1)$$

$$x = 7.77N$$

$$F_y = 1.4 + y = ma_y$$

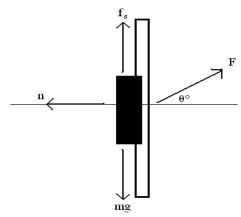
$$F_y = 1.4 + y = (1.7)(-2.5)$$

$$y = -5.65N$$

Since we now have the x and y components of the force, we use Pythagorean equation to get F = 9.61N

#### 19A

A somewhat confusing problem without the correct free-body diagram, shown below:



As always, it's best to put all your force vectors into their x- and y-components (if they aren't already) so we should break apart the applied force, F, into its y-component, Fsin $\theta$ , and its x-component, Fcos $\theta$ .

Now, and here's the important part, since the book isn't sliding down (or moving anywhere really) there's no net force acting on it in either axis/dimension. This basically means that the sum of all the forces pointing up must equal the sum of all those pointing down. And also that the sum of all the forces pointing left equals the sum of those pointing right.

From that we get the following two equations:

$$(1)f_s + F\sin\theta = mg$$

$$(2)n = F\cos\theta$$

The first equation can be re-written to show how friction can be broken down into its constituent parts, and after we substitute n from the second equation, we can then solve for F:

$$f_s + F \sin \theta = mg$$

$$\mu_s n + F \sin \theta = mg$$

$$\mu_s F \cos \theta + F \sin \theta = mg$$

$$F(\mu_s \cos \theta + \sin \theta) = mg$$

$$F = \frac{mg}{\mu_s \cos \theta + \sin \theta} = 5.3N$$

#### 20D

The 100N force will be applied to the system (which has total mass 10kg) causing it to accelerate with  $a = F/m = 100/10 = 10 \text{ m/s}^2$ .

Thus we know that the 6kg mass will be accelerating at 10 m/s<sup>2</sup>, and so the force acting on it must be given by F = ma = (6)(10) = 60N

#### 21E

All of the statements could be true:

- i. If the elevator were accelerating in either direction the normal force would not be equal to the man's weight, however since they don't mention acceleration we must assume the acceleration is zero, and the upward velocity is constant.
- ii. The acceleration would have to be in the upward direction for normal force to be greater, which would mean that although the elevator is moving downward it is slowing down due to an upward acceleration.
- iii. This is always true when accelerating downward.

## 22B

Since the crate is at rest, we would need to overcome the force of static friction in order to get it moving. The maximum force of static friction is given by

$$f_s = \mu_s N = (0.5)(40) = 20N$$

Thus, it is clear that the 12N force applied will not be great enough to overcome static friction and the block will not move. In fact, the force of static friction will simply match the applied force of 12N, ensuring that the object remains still.

#### 23C

Looking at the pulley system, we can generate the equation for the entire system that says the net force acting on the pulley system is:

$$F_{Net} = m_3 g = (m_3 + m_9)a$$
  
 $a = 2.45 \text{ m/s}^2$ 

Now we have acceleration we can set up the equation for net force on either object to solve for tension. For the 3kg mass the equation becomes:

$$F_3 = m_3 g - T = ma$$
$$T = 22.05N$$

#### 24A

This is actually a rather tricky question. Given the information provided we cannot actually calculate the average frictional force, however we do know that since the bear is able to pick up speed during its descent, the weight must be greater than the force of friction. The weight of the bear is mg = 245N, so the average frictional force must be less than 245N, and only one answer obeys that rule.

#### 25C

For a question like this we need to take all the forces acting on the object and put them into the x- and y- components, that way we can set up equations for net force along each axis. In this particular question, we are only really interested in the y-axis, since that is where we'll find the normal force:

The force pointing up will be the normal force, n, and the forces pointing down will be the weight, mg, and the vertical component of the 200N force, 200sin20° = 68.4N

Since the y-axis has no net force, the sum of forces pointing up equal the sum of forces pointing down, thus we set up the equation for the y-axis:

$$F_{Net,y} = 0 : n = mg + 200 \sin 20^{\circ}$$
  
 $n = 313 N$ 

#### 26E

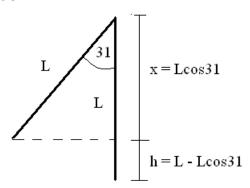
This one's a little tricky. A lot of information is provided, but once they ask for net work and tell us that velocity is constant, we know that  $\Delta K = 0$  and since there is no change in height, the net work done must be zero, since  $\Delta K = W_{Net}$ .

#### 27D

The net force acting on the object will be given by the eqn: Fnet = 100 - f = ma = 3.1 x 20

From that we can solve for the force of friction, f = 38 N

Lastly, Work, 
$$W = -f \times d = -38 \times 20 = -760 \text{ J}$$



This questions really deals with conservation of energy,  $\Delta K + \Delta U = 0$ .

The main problem is determining the initial height, h, of the man just as he starts to swing.

The diagram above shows how the length of the rope, L, can be split into L = x + h, and how we can solve for the height using trig.

So we get the following equation:

$$\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + (0 - mgh) = 0$$
  
 $v_f = 6.77m/s$ 

29E

This is another conservation of energy question, though this time we have friction, so we get

$$\Delta K + \Delta U = Wnc$$

This breaks down to  $\frac{1}{2}mv_f^2 - mgh = -\mu mg\cos 40(6.22)$ , where 6.22 is the distance the block travels down the incline given by (h/sin40).

From this the mass cancels out and v = 6.03 m/s

30D

From the first compression:  $\frac{1}{2}kx^2 = 120$ , so k = 24000 N/m

For the second we just plug in the new k value and solve for x = 0.1207 m

We know that Wnet =  $\Delta K$ ...so if we can get the initial and final velocities we can solve for work.

To do this we must take the derivative of the position function to get v(t).

Now we can calculate the v at t=0 and t=3

$$v(0) = 1 \text{ ms}, v(3) = 10 \text{ m/s}$$

And so: 
$$W = \Delta K = \frac{1}{2}(6)(99) = 297 \text{ J}$$

32C

For this we imagine a number line with the girl at the origin (since the question asks for distance relative to her) at the boy at a distance of 10 m.

To calculate center of mass: 
$$\frac{(40 \times 0) + (60 \times 10)}{100} = 6$$

So we conclude that the center of mass is 6 m from the origin, or 6 m from the girl.

33B

We know that  $\Delta K = Wnet$ 

So we re-write Work into its components, Wnet =  $Fd + -\mu mgd$ 

We then solve for F, and get F = 9.96 N

34C

Power, P is Work/Time. In this question the dog does work by lifting his own weight up the height of the stairs.

So we get: 
$$P = \frac{W}{t} = \frac{mg(15 \times 0.25)}{11} = 83.5W$$

When each child throws the ball he/she gives the ball momentum, p = (0.35)(4.5) = 1.575 kg m/s

Since momentum must be conserved, the child also gets 1.575 kg m/s of momentum, just in the other direction.

Furthermore, after catching the ball, the child picks up another 1.575 kg m/s of momentum.

This means in total each child gets 3.15 kg m/s of momentum.

From this we can calculate the velocity,  $v = \Delta p/m = 0.063$  m/s, where the mass is 490/9.8

36A

Since after the collision the object is now at some angle relative to its initial motion, we have to deal with conservation of momentum in each axis (x and y).

For the x-axis we get:  $10m = (2.5\cos 20)m + (v_x)2m$ , which gives us  $v_x = 3.825$  m/s

For the y-axis we get:  $0 = (2.5\sin 20)m + (v_y)2m$ , which gives us  $v_y = 0.428$  m/s

Note that the masses cancel out in both eqns.

Finally we combine the two velocity vectors to get the resultant velocity,

$$v = \sqrt{{v_x}^2 + {v_y}^2} = 3.85 m/s$$

37B

Another case of conservation of energy:

$$(-\frac{1}{2}mv^2) + (\frac{1}{2}kx^2 - mgh) = 0$$
, where  $h = 6\sin 30 = 3$  m

From that we can solve for initial speed, v = 6.1 m/s

This question is a real pain!

It deals with conservation of energy but this time there are two non-conservative forces that do negative work on the object.

So  $\Delta K + \Delta U =$ Wnc becomes:

$$(-\frac{1}{2}mv^2) + (\frac{1}{2}kx^2) = -F\cos 60(d+x) + -\mu n(d+x)$$

Note that we use (d + x) for distance when calculating work done because the object is still moving as it compresses the spring.

Also, to calculate normal force, n, we use the free body diagram for the y-axis which shows:

$$F\sin 60 + n = mg...$$
and so we get  $n = mg - F\sin 60 = 11.4 \text{ N}$ 

Lastly we solve for  $\mu$  and get 0.64.

39A

The easiest way to do this is to recognize that the two momentums are at right angles and thus their vectors can be combined using Pythagorean theory.

Since momentum is conserved p(initial) = p(final) and we can write:  $(p_x)^2 + (p_y)^2 = p^2$ 

This becomes  $(3m)^2 + (4m)^2 = (v2m)^2$ 

And then  $25m^2 = 4m^2v^2$ , and we get v = 2.5 m/s after the masses cancel out.

This is an equilibrium problem where the net torque has to equal zero. Realize that the upward support forces given by the piers are another way of asking for the normal forces acting on the bridge. Like usual, we should draw a free-body diagram to show the locations of all our forces acting on the bridge and their relative distances from one another. Then we must pick a pivot point before we set up our torque equation. Usually I'd say that it doesn't matter which end you pick for your rotational axis (left or right pier) except that since we are specifically solving for the normal force coming from the right pier we want to make sure that that variable is present in our net torque equation. So we must pick the left pier, to ensure this. Notice then that the weights of the truck and the bridge are both trying to create a clockwise torque about the left-pier pivot point, while the normal force from the right pier is trying to create a counter-clockwise torque. Since the net torque is zero, we know that the clockwise torques are equal to the counter-clockwise torque. (Remember that although the normal force from the left pier is perpendicular to the bridge, it is located at a distance of d = 0 from the pivot point, and so it contributes zero torque about that point).

Finally we set up our equation:

$$(mg)20 + (Mg)25 = (n_{right})50$$
,

where mg is the weight of the truck and Mg is the weight of the bridge.

Since the only unknown variable is n<sub>right</sub> we can solve and get 28.5 kN.