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MATH 251 Exam 2 – Sample Test – Detailed Solutions

1.

$y'' + 9y = 5\sin(2x)$

Find y_c:

$r^2 + 9 = 0$

$r^2 = -9$

$r = \pm 3i$

$y_c = C_1\cos(3x) + C_2\sin(3x)$

Find y_p:

$y_p = A\cos(2x) + B\sin(2x)$

$y_p' = -2A\sin(2x) + 2B\cos(2x)$

$y_p'' = -4A\cos(2x) - 4B\sin(2x)$

$y'' + 9y = 5\sin(2x)$

$-4A\cos(2x) - 4B\sin(2x) + 9A\cos(2x) + 9B\sin(2x) = 5\sin(2x)$

$5A\cos(2x) + 5B\sin(2x) = 0\cos(2x) + 5\sin(2x)$

$A = 0, \quad 5B = 5$

$B = 1$

$y_p = \sin(2x)$

$y = y_c + y_p$

$y = C_1\cos(3x) + C_2\sin(3x) + \sin(2x)$

$y(0) = 0 \rightarrow 0 = C_1$

$y = C_2\sin(3x) + \sin(2x)$

$y' = 3C_2\cos(3x) + 2\cos(2x)$

$y'(0) = 8 \rightarrow 8 = 3C_2 + 2$

$6 = 3C_2$

$C_2 = 2$

$y = 2\sin(3x) + \sin(2x)$

2.

$$y'' + 25y = 3\cos(5x)$$

Find y_c:

$r^2 + 25 = 0$
$r = \pm 5i$

$$y_c = C_1\cos(5x) + C_2\sin(5x)$$

Find y_p:

$$y_p = A\cos(5x) + B\sin(5x)$$ → need to modify

$$y_p = x(A\cos(5x) + B\sin(5x))$$

$$y_p' = x(-5A\sin(5x) + 5B\cos(5x)) + A\cos(5x) + B\sin(5x)$$

$$y_p'' = x(-25A\cos(5x) - 25B\sin(5x)) - 5A\sin(5x) + 5B\cos(5x) - 5A\sin(5x) + 5B\cos(5x)$$

$$= -25Ax\cos(5x) - 25Bx\sin(5x) - 10A\sin(5x) + 10B\cos(5x)$$

$$y'' + 25y = 3\cos(5x)$$

$$-25Ax\cos(5x) - 25Bx\sin(5x) - 10A\sin(5x) + 10B\cos(5x) + 25Ax\cos(5x) + 25Bx\sin(5x) = 3\cos(5x)$$

$$-10A\sin(5x) + 10B\cos(5x) = 3\cos(5x)$$

$$A = 0, \quad B = \frac{3}{10}$$

$$y_p = \frac{3}{10}x\sin(5x)$$

$$y = C_1\cos(5x) + C_2\sin(5x) + \frac{3}{10}x\sin(5x)$$

3.

$m = 2$

$F_s = 16, \; x = 2$

$c = 0$

$$mx'' + cx' + kx = 9\cos(\omega t)$$

Find k:

$F_s = kx$

$16 = k \cdot 2$

$k = 8$

$$2x'' + 8x = 9\cos(\omega t)$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{8}{2}} = 2$$

$$\boxed{\omega = 2}$$

4.

$m = 1$
$c = 0$
$k = 9$
$x(0) = 0$
$x'(0) = 0$

a) $x'' + 9x = 10\cos(3t), \quad x(0) = 0, \ x'(0) = 0$

b) Find x_c:

$r^2 + 9 = 0$
$r = \pm 3i$

$x_c = c_1\cos(3t) + c_2\sin(3t)$

Find x_p:

$x_p = A\cos(3t) + B\sin(3t)$ → need to modify

$x_p = t(A\cos(3t) + B\sin(3t))$

$x_p' = t(-3A\sin(3t) + 3B\cos(3t)) + A\cos(3t) + B\sin(3t)$

$x_p'' = t(-9A\cos(3t) - 9B\sin(3t)) - 3A\sin(3t) + 3B\cos(3t) - 3A\sin(3t) + 3B\cos(3t)$

$= -9At\cos(3t) - 9Bt\sin(3t) - 6A\sin(3t) + 6B\cos(3t)$

→ $x'' + 9x = 10\cos(3t)$

$-9At\cos(3t) - 9Bt\sin(3t) - 6A\sin(3t) + 6B\cos(3t) + 9At\cos(3t) + 9Bt\sin(3t) = 10\cos(3t)$

$-6A\sin(3t) + 6B\cos(3t) = 0\sin(3t) + 10\cos(3t)$

$A = 0, \quad 6B = 10$

$B = \frac{10}{6} = \frac{5}{3}$

$x_p = \frac{5}{3}t\sin(3t)$

$x(t) = x_c + x_p$

$x(t) = c_1\cos(3t) + c_2\sin(3t) + \frac{5}{3}t\sin(3t)$

$x(0) = 0 \rightarrow 0 = C_1 + 0 + 0$

$C_1 = 0$

$$x(t) = C_2 \sin(3t) + \frac{5}{3}t \sin(3t)$$

$$x'(t) = 3C_2 \cos(3t) + \frac{5}{3}t(3\cos(3t)) + \sin(3t) \cdot \frac{5}{3}$$

$x'(0) = 0 \rightarrow 0 = 3C_2 + 0 + 0$

$C_2 = 0$

$$x(t) = \frac{5}{3}t \sin(3t)$$

c) yes

5.

Start by finding the characteristic equation:

$$(1-r)(3-r)-8=0$$
$$3-4r+r^2-8=0$$
$$r^2-4r-5=0$$
$$r=5, r=-1$$

<u>For $r=5$, find the associated k:</u>

$A-rI$

$$\begin{bmatrix} 1-5 & 2 \\ 4 & 3-5 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 4 & -2 \end{bmatrix}$$

So $k_1 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

<u>For $r=-1$, find the associated k:</u>

$A-rI$

$$\begin{bmatrix} 1+1 & 2 \\ 4 & 3+1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}$$

So $k_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$x(t) = C_1 k_1 e^{r_1 t} + C_2 k_2 e^{r_2 t}$$

$$x(t) = C_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{5t} + C_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}$$

6.

Start by finding the characteristic equation:

$$r^2 - 8r + 16 = 0$$
$$(r-4)(r-4) = 0$$
$$r = 4, r = 4$$

For $r = 4$, find the associated k:

$A - rI$

$$\begin{bmatrix} 3-4 & 1 \\ -1 & 5-4 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$$

So $k = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and there is only 1 linearly independent eigenvector. Therefore, the solution will take the form:

$$x(t) = C_1 k e^{rt} + C_2(kte^{rt} + \eta e^{rt})$$

To find η:

$$(A - rI)\eta = k$$

$$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$-1\eta_1 + 1\eta_2 = 1$$

Choose any η_1 and η_2 that satisfy the above equation:

$$\eta_1 = 0, \eta_2 = 1$$

So $\eta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and the overall general solution is:

$$x(t) = C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4t} + C_2 \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} te^{4t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{4t} \right)$$

Use the initial value of $x(0) = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$

$$\begin{bmatrix} 4 \\ -3 \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$4 = C_1$$
$$-3 = C_1 + C_2$$

$$C_2 = -7$$

$$x(t) = 4 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4t} - 7 \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} te^{4t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{4t} \right)$$

$$x(t) = \begin{bmatrix} 4e^{4t} - 7te^{4t} \\ -3e^{4t} - 7t3^{4t} \end{bmatrix}$$

7.

Start by finding the eigenvalues:

$$(2-r)^2+16=0$$
$$(2-r)^2=-16$$
$$2-r=\pm 4i$$
$$r=2\pm 4i$$

For $r = 2+4i$, find the associated k:

$$\begin{bmatrix} 2-(2+4i) & 4 \\ -4 & 2-(2+4i) \end{bmatrix} = \begin{bmatrix} -4i & 4 \\ -4 & -4i \end{bmatrix}$$

So $k_1 = \begin{bmatrix} 4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} i$

$$x(t) = C_1 e^{2t}\left(\begin{bmatrix} 4 \\ 0 \end{bmatrix}\cos(4t) - \begin{bmatrix} 0 \\ 4 \end{bmatrix}\sin(4t)\right) + C_2 e^{2t}\left(\begin{bmatrix} 4 \\ 0 \end{bmatrix}\sin(4t) + \begin{bmatrix} 0 \\ 4 \end{bmatrix}\cos(4t)\right)$$

$$\begin{cases} 3 = 4C_1 \\ 8 = 4C_2 \end{cases}$$

$$C_2 = 3/4$$
$$C_1 = 2$$

$$x(t) = \frac{3}{4}e^{2t}\left(\begin{bmatrix} 4 \\ 0 \end{bmatrix}\cos(4t) - \begin{bmatrix} 0 \\ 4 \end{bmatrix}\sin(4t)\right) + 2e^{2t}\left(\begin{bmatrix} 4 \\ 0 \end{bmatrix}\sin(4t) + \begin{bmatrix} 0 \\ 4 \end{bmatrix}\cos(4t)\right)$$

$$x(t) = \begin{bmatrix} e^{2t}(3\cos(4t) + 6\sin(4t)) \\ e^{2t}(-3\sin(4t) + 6\cos(4t)) \end{bmatrix}$$

8.

Start by finding the eigenvalues:

$$(-2-r)(4-r)+5=0$$
$$-8+2r-4r+r^2+5=0$$
$$r^2-2r-3=0$$
$$(r-3)(r+1)=0$$
$$r_1=3, r_2=-1$$

For $r_1 = 3$, find the associated k_1:

$$\begin{bmatrix} -2-3 & 1 \\ -5 & 4-3 \end{bmatrix} = \begin{bmatrix} -5 & 1 \\ -5 & 1 \end{bmatrix}$$

So $k_1 = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

For $r_2 = -1$, find the associated k_1:

$$\begin{bmatrix} -2+1 & 1 \\ -5 & 4+1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -5 & 5 \end{bmatrix}$$

So $k_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$x(t) = C_1 k_1 e^{r_1 t} + C_2 k_2 e^{r_2 t}$$

$$x(t) = C_1 \begin{bmatrix} 1 \\ 5 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t}$$

$$\begin{cases} 1 = C_1 + C_2 \\ 3 = 5C_1 + C_2 \end{cases}$$

$$3 = 5(1-C_2) + C_2$$
$$3 = 5 - 4C_2$$
$$C_2 = 1/2$$
$$C_1 = 1/2$$

$$x(t) = \frac{1}{2}\begin{bmatrix} 1 \\ 5 \end{bmatrix} e^{3t} + \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t}$$

$$x(t) = \begin{bmatrix} \frac{1}{2}e^{3t} + \frac{1}{2}e^{-t} \\ \frac{5}{2}e^{3t} + \frac{1}{2}e^{-t} \end{bmatrix}$$

9. **Linearly independent since $W \neq 0$**

Solution:

$$W(x_1, x_2) = \begin{bmatrix} 2e^{5t}\cos t & 4e^{5t}\sin t \\ 4e^{5t}\sin t & -8e^{5t}\cos t \end{bmatrix}$$

$W(x_1, x_2) = -16e^{10t}\cos^2 t - 16e^{10t}\sin^2 t = -16e^{10t}(\sin^2 t + \cos^2 t) = -16e^{10t} \neq 0.$

10. **Linearly Dependent since $W = 0$**

$W(x_1, x_2) = \begin{bmatrix} 3e^{-t} & -6e^{-t} \\ -2e^{t} & 4e^{-t} \end{bmatrix} = 12e^{-2t} - 12e^{-2t} = 0$

11.

Manipulate the integral to get in the form $\int_0^\infty e^{-st} f(t)\, dt$, then take the Laplace transform of $f(t)$.

$$\int_0^\infty e^{-st} e^{-4t}\cos(7t)\, dt$$

$$f(t) = e^{-4t}\cos(7t)$$

$$\mathcal{L}\{e^{-4t}\cos(7t)\} = \frac{s+4}{(s+4)^2 + 49}$$

12.

$$1 + u(t-1)(t^2-1) + u(t-2\pi)(\cos t - t^2)$$

$$\mathcal{L}\{1\} + \mathcal{L}\{u(t-1)(t^2-1)\} + \mathcal{L}\{u(t-2\pi)(\cos t - t^2)\}$$

$$\frac{1}{s} + e^{-s}\,\mathcal{L}\{(t+1)^2 - 1\} + e^{-2\pi s}\mathcal{L}\{\cos(t+2\pi) - (t+2\pi)^2\}$$

$$\frac{1}{s} + e^{-s}\,\mathcal{L}\{t^2 + 2t + 1 - 1\} + e^{-2\pi s}\mathcal{L}\{\cos(t) - (t^2 + 4\pi t + 4\pi^2)\}$$

$$\frac{1}{s} + e^{-s}\,\mathcal{L}\{t^2 + 2t\} + e^{-2\pi s}\mathcal{L}\{\cos(t) - t^2 - 4\pi t - 4\pi^2\}$$

$$\frac{1}{s} + e^{-s}\left(\frac{2}{s^3} + \frac{2}{s^2}\right) + e^{-2\pi s}\left(\frac{s}{s^2+1} - \frac{2}{s^3} - \frac{4\pi}{s^2} - \frac{4\pi^2}{s}\right)$$

13.

a. Use Partial Fractions, then take the inverse Laplace.

$$\frac{36-s}{s(s+6)^2} = \frac{A}{s} + \frac{B}{(s+6)} + \frac{C}{(s+6)^2}$$

$$36 - s = A(s+6)^2 + Bs(s+6) + Cs$$

$$s = 0 \rightarrow A = 1$$
$$s = -6 \rightarrow C = -7$$

$$36 - s = 1(s+6)^2 + Bs(s+6) - 7s$$

$$36 - s = s^2 + 12s + 36 + Bs^2 + 6Bs - 7s$$

Equating the coefficients gives us $1 + B = 0$, so $B = -1$

$$\frac{36-s}{s(s+6)^2} = \frac{1}{s} - \frac{1}{(s+6)} - \frac{7}{(s+6)^2}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{(s+6)} - \frac{7}{(s+6)^2}\right\} = \mathbf{1 - e^{-6t} - 7e^{-6t}t}$$

b. Since the denominator does not factor, start by completing the square.

$$\frac{2s-2}{s^2 + 2s + \mathbf{1} + 17 - \mathbf{1}}$$

$$2\frac{s-1}{(s+1)^2 + 16}$$

$$2\frac{s + \mathbf{1} - 1 - \mathbf{1}}{(s+1)^2 + 16}$$

$$2\left[\frac{s+1}{(s+1)^2 + 16} - \frac{2}{(s+1)^2 + 16}\right]$$

Now take the inverse Laplace

$$2\mathcal{L}^{-1}\left[\frac{s+1}{(s+1)^2 + 16} - \frac{1}{\mathbf{2}}\frac{2 * \mathbf{2}}{(s+1)^2 + 16}\right]$$

$$= 2\left[e^{-t}\cos(4t) - \frac{1}{2}e^{-t}\sin(4t)\right]$$

c. Take inverse Laplace of each term separately. You will have to complete the square for the first term...

$$\mathcal{L}^{-1}\left\{e^{-3s}\frac{s}{s^2+6s+\mathbf{9}+25-\mathbf{9}}-5e^{-4s}\right\}$$

$$\mathcal{L}^{-1}\left\{e^{-3s}\frac{s}{(s+3)^2+16}-5e^{-4s}\right\}$$

$$\mathcal{L}^{-1}\left\{e^{-3s}\frac{s\mathbf{+3-3}}{(s+3)^2+16}-5e^{-4s}\right\}$$

$$\mathcal{L}^{-1}\left\{e^{-3s}\left(\frac{s+3}{(s+3)^2+16}-\frac{3}{(s+3)^2+16}\right)-5e^{-4s}\right\}$$

$$\mathcal{L}^{-1}\left\{e^{-3s}\left(\frac{s+3}{(s+3)^2+16}-\frac{3}{(s+3)^2+16}\right)\right\}-\mathcal{L}^{-1}\{5e^{-4s}\}$$

$$=u(t-3)\mathcal{L}^{-1}\left\{\left(\frac{s+3}{(s+3)^2+16}-\frac{3}{(s+3)^2+16}\right)\Bigg|_{t=t-3}\right\}-5\delta(t-4)$$

$$=u(t-3)\mathcal{L}^{-1}\left\{\left(\frac{s+3}{(s+3)^2+16}-\frac{\mathbf{1}}{\mathbf{4}}\frac{3\mathbf{(4)}}{(s+3)^2+16}\right)\Bigg|_{t=t-3}\right\}-5\delta(t-4)$$

$$=u(t-3)\left[e^{-3(t-3)}\cos\big(4(t-3)\big)-\frac{3}{4}e^{-3(t-3)}\sin\big(4(t-3)\big)\right]-5\delta(t-4)$$

14. Start by finding the Laplace transform of both sides.

$$\mathcal{L}\{y''\} + 4\mathcal{L}\{y'\} + 3\mathcal{L}\{y\} = \mathcal{L}\{2\delta(t-1)\}$$

$$s^2Y - s\cdot y(0) - y'(0) + 4[sY - y(0)] + 3Y = 2e^{-s}$$

$$s^2Y - 2 + 4sY + 3Y = 2e^{-s}$$

$$s^2Y + 4sY + 3Y = 2e^{-s} + 2$$

$$(s^2 + 4s + 3)Y = 2e^{-s} + 2$$

$$Y = e^{-s}\frac{2}{(s+3)(s+1)} + \frac{2}{(s+3)(s+1)}$$

$$y = \mathcal{L}^{-1}\left\{e^{-s}\frac{2}{(s+3)(s+1)} + \frac{2}{(s+3)(s+1)}\right\}$$

Partial Fraction Decomposition:

$$\frac{2}{(s+3)(s+1)} = \frac{A}{s+3} + \frac{B}{s+1} \qquad \rightarrow A = -1, B = 1$$

Back to the equation:

$$y = \mathcal{L}^{-1}\left\{e^{-s}\left[\frac{-1}{s+3} + \frac{1}{s+1}\right]\right\} + \mathcal{L}^{-1}\left\{\frac{-1}{s+3} + \frac{1}{s+1}\right\}$$

$$y = u(t-1)\mathcal{L}^{-1}\left\{\frac{-1}{s+3} + \frac{1}{s+1}\bigg|_{t=t-1}\right\} + \mathcal{L}^{-1}\left\{\frac{-1}{s+3} + \frac{1}{s+1}\right\}$$

$$y = u(t-1)\left(-e^{-3(t-1)} + e^{-(t-1)}\right) - e^{-3t} + e^{-t}$$

15. Start by finding the Laplace transform of both sides.

$$\mathcal{L}\{y''\} - 9\mathcal{L}\{y\} = \mathcal{L}\{u(t-6)\} + \mathcal{L}\{\delta(t-3)\}$$

$$s^2Y - s \cdot y(0) - y'(0) - 9Y = \frac{e^{-6s}}{s} + e^{-3s}$$

$$s^2Y - 9Y = \frac{e^{-6s}}{s} + e^{-3s}$$

$$Y(s^2 - 9) = \frac{e^{-6s}}{s} + e^{-3s}$$

$$Y = \frac{e^{-6s}}{s(s+3)(s-3)} + \frac{e^{-3s}}{(s+3)(s-3)}$$

$$y = \mathcal{L}^{-1}\left\{e^{-6s}\frac{1}{s(s+3)(s-3)}\right\} + \mathcal{L}^{-1}\left\{e^{-3s}\frac{1}{(s+3)(s-3)}\right\}$$

PFD #1:

$$\frac{1}{s(s+3)(s-3)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s-3}$$

$$A = -\frac{1}{9}, B = \frac{1}{18}, C = \frac{1}{18}$$

PFD #2:

$$\frac{1}{(s+3)(s-3)} = \frac{A}{s+3} + \frac{B}{s-3}$$

$$A = -\frac{1}{6}, B = \frac{1}{6}$$

$$y = \mathcal{L}^{-1}\left\{e^{-6s}\left(-\frac{1}{9}\cdot\frac{1}{s} + \frac{1}{18}\cdot\frac{1}{s+3} + \frac{1}{18}\cdot\frac{1}{s-3}\right)\right\} + \mathcal{L}^{-1}\left\{e^{-3s}\left(-\frac{1}{6}\cdot\frac{1}{s+3} + \frac{1}{6}\cdot\frac{1}{s-3}\right)\right\}$$

$$y = u(t-6)\,\mathcal{L}^{-1}\left(-\frac{1}{9}\cdot\frac{1}{s} + \frac{1}{18}\cdot\frac{1}{s+3} + \frac{1}{18}\cdot\frac{1}{s-3}\right)\Bigg|_{t=t-6} + u(t-3)\mathcal{L}^{-1}\left(-\frac{1}{6}\cdot\frac{1}{s+3} + \frac{1}{6}\cdot\frac{1}{s-3}\right)\Bigg|_{t=t-3}$$

$$y = u(t-6)\left(-\frac{1}{9} + \frac{1}{18}e^{-3(t-6)} + \frac{1}{18}e^{3(t-6)}\right) + u(t-3)\left(-\frac{1}{6}e^{-3(t-3)} + \frac{1}{6}e^{3(t-3)}\right)$$

16.

a. True

b. False – $A = \sqrt{(C_1)^2 + (C_2)^2} = \sqrt{2^2 + 2^2} = \sqrt{8}$

c. False – It will remain constant.

17.

a.

$$r^2 + 4r + 9 = 0$$

$$r = -2 \pm \sqrt{5}i$$

$$x(t) = C_1 e^{-2t}\cos(\sqrt{5}t) + C_2 e^{-2t}\sin(\sqrt{5}t)$$

$$x(0) = 0:$$
$$0 = C_1\cos(0) + C_2\sin(0)$$
$$C_1 = 0$$

$$x(t) = C_2 e^{-2t}\sin(\sqrt{5}t)$$

$$x'(t) = \sqrt{5}C_2 e^{-2t}\cos(\sqrt{5}t) - 2C_2 e^{-2t}\sin(\sqrt{5}t)$$

$$x'(0) = 1:$$
$$1 = \sqrt{5}C_2\cos(0) - 2C_2\sin(0)$$
$$C_2 = 1/\sqrt{5}$$

$$x(t) = \frac{1}{\sqrt{5}}e^{-2t}\sin(\sqrt{5}t)$$

b. $m = 1, \gamma = 4, k = 9$

Underdamped

c.

$$x = 0$$

$$0 = \frac{1}{\sqrt{5}}e^{-2t}\sin(\sqrt{5}t)$$

$$0 = \sin(\sqrt{5}t)$$

$$\sqrt{5}t = n\pi$$

$$t = \frac{n\pi}{\sqrt{5}}$$

18.

$$m = 1, L = \frac{5}{2}, \gamma = 0, \quad x(0) = 1, x'(0) = 0$$

a. Use Hooke's Law:

$$mg = KL$$
$$(1)(10) = k\left(\frac{5}{2}\right)$$
$$k = 4$$

$$x'' + 4x = 0, x(0) = 1, x'(0) = 0$$

b.

$$r^2 + 4 = 0$$

$$r^2 = \pm 2i$$

$$x(t) = C_1 \cos(2t) + C_2 \sin(2t)$$

$$x(0) = 1:$$
$$1 = C_1 \cos(0) + C_2 \sin(0)$$
$$C_1 = 1$$

$$x(t) = \cos(2t) + C_2 \sin(2t)$$

$$x'(t) = -2\sin(2t) + 2C_2 \cos(2t)$$

$$x'(0) = 0:$$
$$0 = -2\sin(0) + 2C_2 \cos(0)$$
$$0 = C_2$$

$$x(t) = \cos(2t)$$

c. The system is undamped, so the lowest point is equal to the amplitude.

$$A = \sqrt{(C_1)^2 + (C_2)^2} = \sqrt{1^2 + 0^2} = 1\, m \text{ below the equilibrium position}$$

d.

$$x(t) = 0$$

$$\cos(2t) = 0$$

$$2t = \frac{(2n+1)\pi}{2}$$

$$t = \frac{(2n+1)\pi}{4}$$