LionTutors® Practice Exam

MATH 140 - Final (New Material)

1. A. Use the Fundamental Theorem of Calculus:

$$(\tan^{-1}(2)) \cdot (0) - (\tan^{-1}(x^3)) \cdot 3x^2$$

$$-3x^2 \tan^{-1}(x^3)$$

2. D. FOIL the numerator, and then divide by x to simplify before integrating:

$$\int \left(\frac{x^2}{x} + \frac{2x}{x} + \frac{1}{x} \right) dx$$

$$\int (x + 2 + x^{-1})\, dx$$

$$\frac{x^2}{2} + 2x + \ln x \Big]_2^8$$

$$\left(\frac{64}{2} + 16 + \ln 8 \right) - \left(\frac{4}{2} + 4 + \ln 2 \right)$$

$$48 + \ln 8 - 6 - \ln 2$$

$$42 + \underbrace{\ln 8 - \ln 2}$$

$$\ln 8 - \ln 2 = \ln\left(\frac{8}{2}\right) = \ln 4$$

$$42 + \ln 4$$

3. C. Use the Net Change Theorem

$$\int_0^5 (250 - 10t)\, dt$$

$$= 250t - 5t^2 \Big]_0^5$$

$$= \left(250(5) - 5(25) \right) - 0$$

$= 1125$ liters

4. A
5. C
6. B

7. Evaluate each integral using u-substitution:

a. $$\int \frac{e^x}{(1-e^x)^2}\, dx$$

$u = 1 - e^x$

$du = -e^x dx$

$dx = \frac{du}{-e^x}$

$$= \int \frac{\cancel{e^x}}{u^2} \frac{du}{-\cancel{e^x}}$$

$$= \int -u^{-2} du$$

$$= -\frac{u^{-1}}{-1} = \frac{1}{u} = \boxed{\frac{1}{1-e^x} + C}$$

b. $\int_1^2 x\sqrt{x-1}\,dx$

$u = x-1$
$du = dx$

$\int_{-}^{-} x \cdot u^{1/2}\,du \longrightarrow$ $u = x-1$
$x = u+1$

$\int (u+1)u^{1/2}\,du$

$\int \left(u^{3/2} + u^{1/2}\right) du$

$\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}$

$\frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} \Big]_1^2$

$\left[\frac{2}{5}(1)^{5/2} + \frac{2}{3}(1)^{3/2}\right] - \left[0 + 0\right]$

$\frac{2}{5} + \frac{2}{3} = \frac{6}{15} + \frac{10}{15} = \boxed{\frac{16}{15}}$

c. $\int \tan(3x)\,dx$

$= \int \frac{\sin(3x)}{\cos(3x)}\,dx$

$u = \cos(3x)$

$du = -3\sin(3x)\,dx$

$dx = \frac{du}{-3\sin(3x)}$

$= \int \frac{\cancel{\sin}(3x)}{u} \cdot \frac{du}{-3\cancel{\sin}(3x)}$

$= \int -\frac{1}{3}u^{-1}\,du$

$= -\frac{1}{3}\ln|u| + C$

$= \boxed{-\frac{1}{3}\ln|\cos(3x)| + C}$

d.

$$\int \frac{2x+2}{x^2+4}\,dx$$

$$\underbrace{\int \frac{2x}{x^2+4}\,dx}_{\downarrow} + \underbrace{\int \frac{2}{x^2+4}\,dx}_{\downarrow}$$

$u = x^2+4$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$$\int 2\cdot\frac{1}{x^2+4}\,dx$$

$$\int \frac{2x}{u}\frac{du}{2x}$$

$$2\left(\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right)$$

$$\int u^{-1}$$

$$\tan^{-1}\left(\frac{x}{2}\right)$$

$$\ln|u|$$

$$\ln(x^2+4)$$

$$\boxed{\ln(x^2+4) + \tan^{-1}\left(\frac{x}{2}\right) + C}$$

8. Use u-substitution for each of the integrals:

a. $\int_0^2 (5 + f'(4x))dx$

$= \int_0^2 5\,dx + \int_0^2 f'(4x)\,dx$

$u = 4x$

$du = 4dx$

$\int f'(u) \frac{du}{4}$

$= \frac{1}{4}\int f'(u)du$

$= \frac{1}{4} f(u)$

$= \frac{1}{4} f(4x)$

$= 5x\Big]_0^2 + \frac{1}{4} f(4x)\Big]_0^2$

$= [5(2) - 5(0)] + [\frac{1}{4} f(8) - \frac{1}{4} f(0)]$

$= 10 + \frac{1}{4}(0) - \frac{1}{4}(1)$

$= 10 - \frac{1}{4} = \boxed{\frac{39}{4}}$

b. $\int_0^5 f'(2x)e^{f(2x)}dx$

$u = f(2x)$
$du = f'(2x)\cdot 2\,dx$
$dx = \dfrac{du}{2f'(2x)}$

$\int \cancel{f'(2x)}\, e^u \dfrac{du}{2\cancel{f'(2x)}}$

$\int \frac{1}{2}e^u\,du \quad = \quad \frac{1}{2}e^u \quad = \quad \frac{1}{2}e^{f(2x)}\Big]_0^5$

$= \frac{1}{2}e^{f(10)} - \frac{1}{2}e^{f(0)} \quad = \quad \frac{1}{2}e^5 - \frac{1}{2}e^1 \quad = \quad \boxed{\dfrac{e^5 - e}{2}}$

c. $\int_0^8 \dfrac{f'(x)}{1+(f(x))^2}\,dx$

$u = f(x)$
$du = f'(x)dx$
$dx = \dfrac{du}{f'(x)}$

$\int \dfrac{\cancel{f'(x)}}{1+u^2}\,\dfrac{du}{\cancel{f'(x)}}$

$\int \dfrac{1}{1+u^2}\,du$

$\tan^{-1}(u)$

$\tan^{-1}(f(x))\Big]_0^8$

$\tan^{-1}(f(8)) - \tan^{-1}(f(0))$

$\tan^{-1}(0) - \tan^{-1}(1) = 0 - \frac{\pi}{4} = \boxed{-\frac{\pi}{4}}$

d. $\displaystyle\int_0^2 \frac{f'(5x)}{f(5x)}\,dx$

$u = f(5x)$

$du = f'(5x) \cdot 5\,dx$

$dx = \dfrac{du}{5f'(5x)}$

$\displaystyle\int \frac{\cancel{f'(5x)}}{u} \cdot \frac{du}{5\cancel{f'(5x)}}$

$\displaystyle\int \frac{1}{5}u^{-1}\,du$

$\frac{1}{5}\ln|u|$

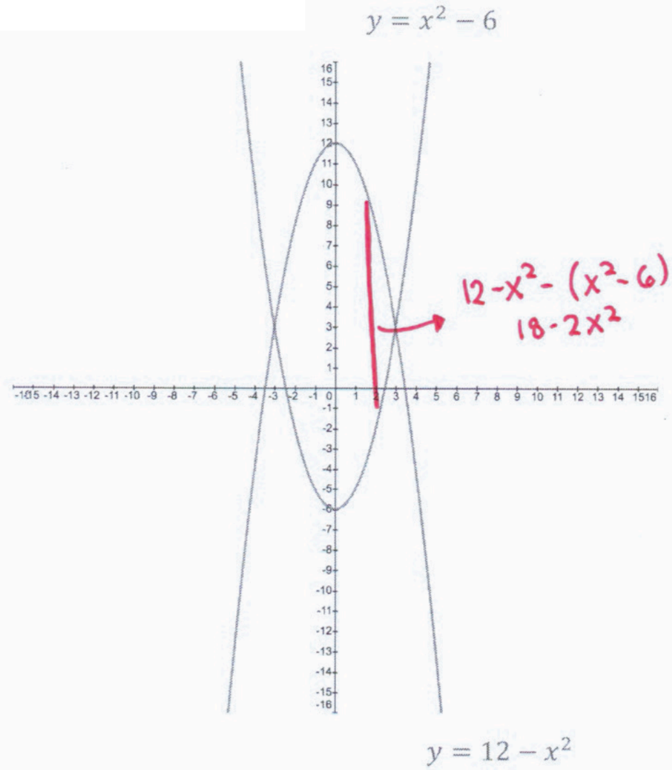
$\frac{1}{5}\ln|f(5x)|\Big]_0^2$

$\frac{1}{5}\ln|f(10)| - \frac{1}{5}\ln|f(0)|$

$\frac{1}{5}\ln(5) - \cancel{\frac{1}{5}\ln(1)}^{\,0}$

$\boxed{\frac{1}{5}\ln(5)}$

9.



a) First, find the intersection points:

$12 - x^2 = x^2 - 6$

$2x^2 = 18$

$x^2 = 9$

$x = -3, x = 3$

To find the area, evaluate the integral:

$$\int_{-3}^{3} (12 - x^2 - [x^2 - 6])\, dx$$

$$\int_{-3}^{3} (18 - 2x^2)\, dx$$

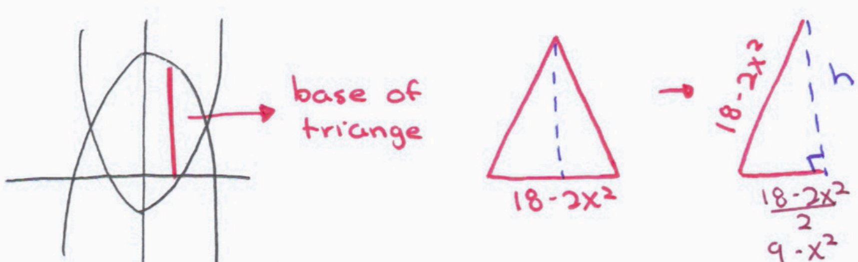
or by symmetry you can evaluate:

$$2\int_{0}^{3} (18 - 2x^2)\, dx$$

$$2\left[18x - \frac{2}{3}x^3\right]_0^3$$

$$2[54 - 18] = 72$$

b) To find the volume with equilateral triangles as cross sections:



$(9-x^2)^2 + h^2 = (18-2x^2)^2$

$h^2 = (18-2x^2)^2 - (9-x^2)^2$

$h^2 = 324 - 72x^2 + 4x^4 - 81 + 18x^2 - x^4$

$h^2 = 243 - 54x^2 + 3x^4$

$h = \sqrt{3x^4 - 54x^2 + 243}$

$h = \sqrt{3(x^4 - 18x^2 + 81)}$

$h = \sqrt{3(x^2-9)^2}$

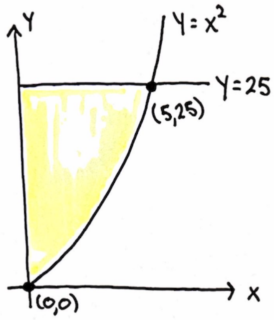
$h = \sqrt{3}(x^2-9)$

$A_\Delta = \frac{1}{2}bh = \frac{1}{2}(18-2x^2)\sqrt{3}(x^2-9)$

$$\boxed{V = \frac{\sqrt{3}}{2}\int_{-3}^{3} (18-2x^2)(x^2-9)\, dx}$$

10.

a)



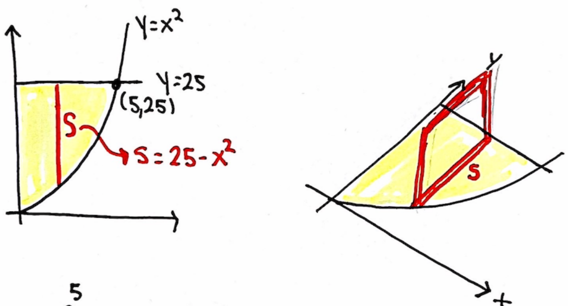
Int. Points.

$x^2 = 25$

$x = \pm 5$, but only use $+5$ since we are in Q1.

$$A = \int_0^5 (25 - x^2)\, dx$$

b)

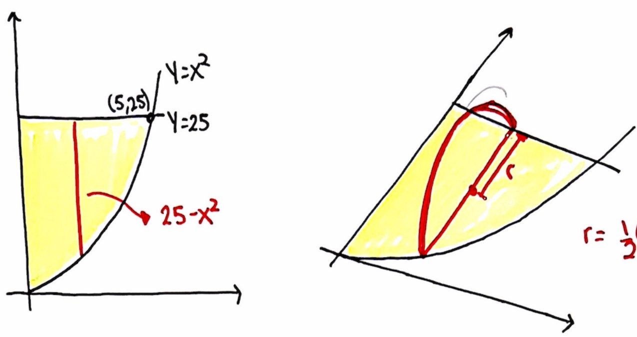


$A_s = (S)^2$

$A_s = (25 - x^2)^2$

$$V = \int_0^5 (25 - x^2)^2\, dx$$

c)



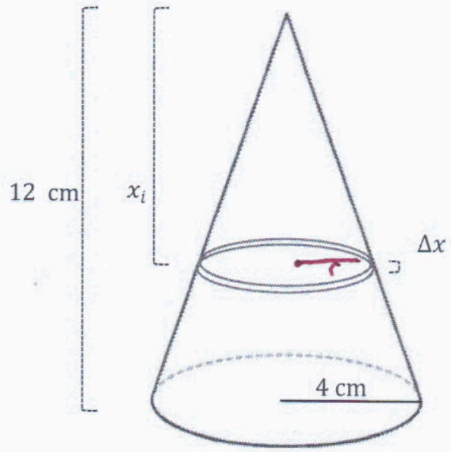
$r = \frac{1}{2}(25 - x^2)$

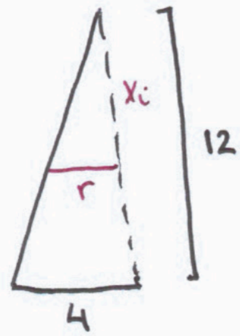
$A = \frac{1}{2}\pi r^2$

$A = \frac{1}{2}\pi \left(\frac{1}{2}(25 - x^2)\right)^2 = \frac{1}{8}\pi (25 - x^2)^2$

$$V = \int_0^5 \frac{1}{8}\pi (25 - x^2)^2\, dx$$

11.





$$\frac{x_i}{12} = \frac{r}{4}$$

$$4x_i = 12r$$

$$r = \frac{4x_i}{12}$$

$$r = \frac{x_i}{3}$$

a) $A_{slice} = \pi r^2$

$$= \pi\left(\frac{x_i}{3}\right)^2$$

$V_{slice} = \pi\left(\frac{x_i}{3}\right)^2 \Delta x$

b) $V = \int_0^{12} \pi\left(\frac{x}{3}\right)^2 dx$

12.

Calculate the work for the cable and coal separately, then add them together.

Coal:

$W = Fd$

$W = (80)(300)$

$W = 24{,}000$

Cable:

$f(x) = 0.2x$

$$W = \int_0^{300} 0.2x\,dx$$

$W = [0.1x^2]_0^{300}$

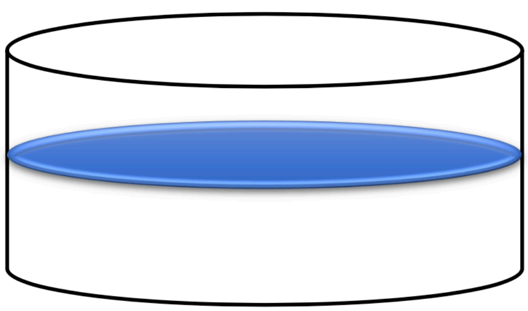
$W = 0.1(300)^2$

$W = 0.1(90{,}000)$

$W = 9{,}000$

Total Work = 33,000 foot-pounds

13.



$V_s = \pi r^2 h$

$V_s = \pi(10)^2\Delta x = 100\pi\Delta x$

$m_s = (density) \times (volume)$

$m_s = (1000)(\ 100\pi\Delta x) = 100{,}000\pi\Delta x$

$F_s = mg$

$F_s = (9.8)(100{,}000)\pi\Delta x$

$W_s = Fd$

$W_s = (9.8)(100{,}000)\pi\Delta x(x)$

$W_s = (9.8)(100{,}000)\pi x\Delta x$

$$W = \int_0^4 (9.8)(100{,}000)\pi x\,dx$$

$$W = (9.8)(100{,}000)\left(\frac{1}{2}x^2\right)\pi\Big]_0^4$$

$$W = (9.8)(100{,}000)\left(\frac{1}{2}\right)(16)\pi$$

$\boldsymbol{W = (9.8)(800{,}000)\pi}$ **J**

14.

Use Hooke's Law to find $f(x)$:

$$f(x) = kx$$
$$20 = k5$$
$$k = 4$$
$$f(x) = 4x$$

Now use the formula for work:

$$W = \int_a^b f(x)\,dx$$

$$W = \int_0^3 4x\,dx = 2x^2]_0^3 = 2(9) - 2(0) = 18 \text{ ft lbs}$$

15.

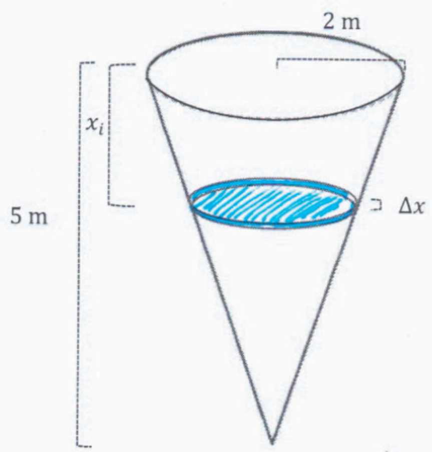
$$W = \int_a^b f(x)\,dx$$

$$W = \int_0^2 x^2 e^{x^3}\,dx$$

$$W = \left[\frac{1}{3}e^{x^3}\right]_0^2$$

$$W = \frac{1}{3}e^8 - \frac{1}{3} = \frac{e^8 - 1}{3} \text{ ft lbs}$$

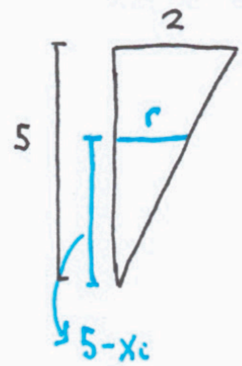
16.



$V_{slice} = \pi r^2 h$

$V_{slice} = \pi r^2 \Delta x$

Find r:



$\frac{5}{2} = \frac{5-x_i}{r}$

$5r = 2(5-x_i)$

$r = \frac{2}{5}(5-x_i)$

$V_{slice} = \pi\left(\frac{2}{5}(5-x_i)\right)^2 \Delta x = \frac{4}{25}\pi(5-x_i)^2 \Delta x$

a)

$W_{slice} = F \cdot d$

$F_{slice} = \text{Vol} \times \rho g$

$F_{slice} = \frac{4}{25}\pi(5-x_i)^2 \Delta x \, \rho g$

$$W_{slice} = \underbrace{\frac{4}{25}\pi \rho g (5-x_i)^2 \Delta x}_{F_{slice}} \cdot \underbrace{x_i}_{\text{distance traveled by slice}}$$

b)

$$W = \int_0^5 \frac{4}{25}\pi(1000)(9.8)(5-x)^2 x \, dx$$