www.LionTutors.com

MATH110 – Final Exam (Review of Exam 1, 2, & 3) – Sample Test Solutions

1. D

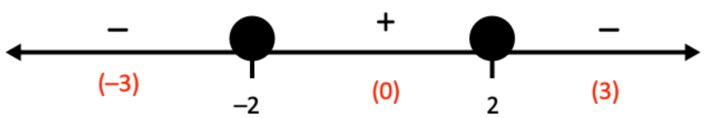
To find the domain, we need to look at the radical and the denominator. Whatever is underneath the radical must be ≥ 0 and the denominator ≠ 0. Since we have an x^2 underneath the radical, we will need to draw out a number line to test points. Positive segments on the number line are in the domain.

Radical

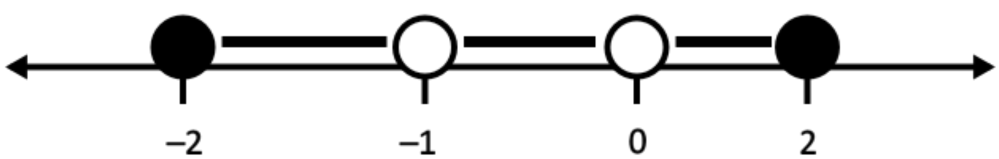
$$4-x^2 \geq 0$$
$$(2-x)(2+x) \geq 0$$
$$x=2, x=-2$$

Denominator

$$x^2+x \neq 0$$
$$x(x+1) \neq 0$$
$$x \neq 0, x \neq -1$$



Putting those together gives us a domain of $[-2,-1) \cup (-1,0) \cup (0,2]$



2. B

Start by plugging in -4. This will give you 0/0, so you must simplify by factoring. Once you simplify and cross something out, plug -4 back in for x.

$$\lim_{x\to -4} \frac{x^2-16}{x^3+4x^2}$$

$$\lim_{x\to -4} \frac{(x-4)(x+4)}{x^2(x+4)}$$

$$\lim_{x\to -4} \frac{(x-4)}{x^2} = \frac{-4-4}{16} = \frac{-8}{16} = -\frac{1}{2}$$

3. A

We will need to use point-slope form to find the equation of the tangent line. Therefore, we need a point and a slope. We are given the point $(1, -2)$.

To find the slope of the tangent line take the derivative, then plug in 1. You will need to use a product rule to take the derivative of the first term.

$$y' = e^x \frac{1}{x} + \ln x (e^x)$$

$$y'(1) = e^1 \frac{1}{1} + \ln(1)(e^1) = e + 0 = e$$

$$m = e$$

Once you have the slope, plug into point-slope form.

$$y - (-2) = e(x - 1)$$

$$y + 2 = ex - e$$

$$y = ex - e - 2$$

4. C

The first thing to do is solve for x in order to find $f(p)$ and $f'(p)$. Then, find the elasticity by using the elasticity formula.

$$3x + 9p = 15$$

$$3x = 15 - 9p$$

$$x = 5 - 3p$$

$$f(p) = 5 - 3p$$

$$E = \frac{-p \times f'(p)}{f(p)}$$

$$E = \frac{-1(-3)}{5 - 3(1)}$$

$$E = \frac{3}{2} = 1.5$$

By definition, since $E > 1$, it is elastic.

5. E

First, use the chain rule to take the first derivative. Then, simplify before taking the second derivative:

$$f'(x) = \frac{5}{2}(4x^2 - 1)^{\frac{3}{2}}(8x)$$
$$f'(x) = (20x)(4x^2 - 1)^{\frac{3}{2}}$$

Now, take the second derivative. We must use the product rule overall, chain rule within. Then, simplify the second derivative:

$$f''(x) = (20x) \cdot \frac{3}{2}(4x^2 - 1)^{\frac{1}{2}}(8x) + (4x^2 - 1)^{\frac{3}{2}}(20)$$
$$f''(x) = 240x^2(4x^2 - 1)^{\frac{1}{2}} + 20(4x^2 - 1)^{\frac{3}{2}}$$

Now, factor and simplify:

$$f''(x) = 20(4x^2 - 1)^{\frac{1}{2}}(12x^2 + 4x^2 - 1)$$
$$f''(x) = 20\sqrt{4x^2 - 1}(16x^2 - 1)$$

6. **C**

The first thing that you want to do is remember that critical points must be in the domain of the original function. Since the denominator cannot be 0, $x \neq 0$. Eliminate answer choices A and B right away.

Then, take the first derivative using the quotient rule. After taking the derivative, factor the numerator:

$$f'(x) = \frac{x2e^x - 2e^x}{x^2} = \frac{2e^x(x-1)}{x^2}$$

When you set the top equal to 0, you get $x = 1$. Remember that all exponential functions are positive, so an exponential function will never be equal to 0. When you set the bottom equal to 0, you get $x = 0$, however that is not in the domain of the original function, so the only critical number is $x = 1$.

7. B

First, we must remember that relative extrema must occur at critical points, and critical points must be in the domain of the original function. We have a natural log here, so we must make sure that *whatever is inside the is positive!*

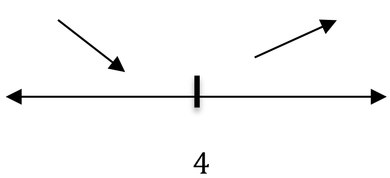
$5x > 0$
$x > 0$

Since we know that x must be greater than 0, we can eliminate answer choices C, D, and E. Since relative extrema must occur at a critical point, we cannot have a relative extremum at a value that is not a critical point.

Now, we must find the function's critical points by taking the derivative of the function, setting the top and bottom of the derivative equal to zero, and solving for x.

$$f'(x) = 4x - 64\left(\frac{1}{x}\right)$$
$$f'(x) = 4x - \frac{64}{x}$$
$$f'(x) = 4x\left(\frac{x}{x}\right) - \frac{64}{x}$$
$$f'(x) = \frac{4x^2 - 64}{x} = \frac{4(x^2 - 16)}{x} = \frac{4(x+4)(x-4)}{x}$$

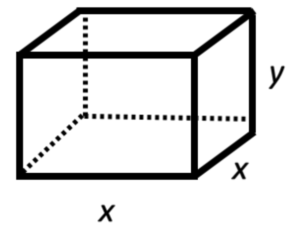
Setting the top and bottom equal to 0 gives us $x = -4, x = 4$, and $x = 0$. However, we've already established that $x > 0$. Therefore, $x = 4$ is the only critical point. Now we can create a first derivative number line to find the functions relative extrema.



Since the function goes from decreasing to increasing at 4, $x = 4$ is a relative minimum and there is no relative maximum.

8. A

This is an optimization question. You want to minimize cost, so you need to create a function for C(x). Sketch a picture of the box to help you put together your cost equation.



$C(x) = 5x^2 + 1x^2 + 4(3)xy$

The function must be in the variable *x*, so you will need to use the volume equation to find *y* in terms of *x*.

$V = 8$
$x^2y = 8$
$y = \frac{8}{x^2}$

Now, plug back in to the cost function to get everything in terms of the variable *x*. Then, simplify and put into derivative friendly format.

$C(x) = 6x^2 + 12x\left(\frac{8}{x^2}\right)$
$C(x) = 6x^2 + 96x^{-1}$

Now, take the derivative of the cost function, set the derivative equal to zero, and solve for *x*.

$C'(x) = 12x - 96x^{-2}$
$0 = 12x - \frac{96}{x^2}$
$\frac{96}{x^2} = 12x$
$12x^3 = 96$
$x^3 = 8$
$x = 2$

9. B

Use logarithmic differentiation. Don't forget to use the product rule when you take the derivative.

$$y = x^x$$
$$\ln y = \ln x^x$$
$$\ln y = x \ln x$$
$$\frac{1}{y} y' = x\frac{1}{x} + \ln x$$

$$y' = (1 + \ln x) y$$
$$y' = (1 + \ln x) x^x$$
$$y'(3) = (1 + \ln 3) 27$$

10. D

Here, we are asked to solve for t, so the first thing that we must do is use algebra to get the term with the t alone.

$$54 = 27(1 + 3e^{-0.1t})$$
$$2 = 1 + 3e^{-0.1t}$$
$$1 = 3e^{-0.1t}$$
$$\frac{1}{3} = e^{-0.1t}$$

Now that we've isolated the term with the t, we must take the natural log of both sides. Then, simplify with the rules of logs.

$$\ln\left(\frac{1}{3}\right) = ln(e^{-0.1t})$$
$$\ln\left(\frac{1}{3}\right) = -0.1t$$
$$\ln\left(\frac{1}{3}\right) = \frac{-1}{10} t$$
$$t = -10 \ln\left(\frac{1}{3}\right)$$

Since the answer above does not match with an answer choice, we must simplify further using the rules of logs.

$$t = -10 \ln\left(\frac{1}{3}\right) = -10[\ln(1) - \ln(3)]$$
$$t = -10[0 - \ln(3)] = 10\ln(3)$$

11. E

This is an optimization question. Start by finding the profit equation, then take the derivative and set it equal to 0.

$$P(x) = px - C$$
$$P(x) = (250 - x)x - (10x + 500)$$
$$P(x) = 250x - x^2 - 10x - 500$$
$$P(x) = 240x - x^2 - 500$$
$$P'(x) = 240 - 2x$$
$$0 = 240 - 2x$$
$$2x = 240$$
$$x = 120$$
$$p(x) = 250 - 120 = \$130$$

12. B

Since the question is asking you to find the limiting value of cost when the number of units grows infinitely large, what they are really asking you to do is to the limit as $x \to \infty$

$$\lim_{x\to\infty}\left[80 - \frac{5x^3 + 2x - 1}{x^3 + 1} + \frac{200(x+1)^2}{x^4 + 2x - 1}\right] = 80 - 5 + 0 = 75$$

To evaluate this limit, you want to look at the highest power on top and bottom in each term of the Cost equation.

13. D

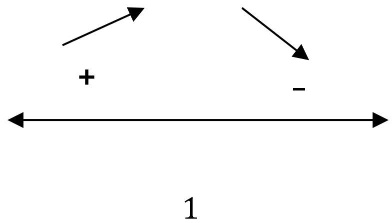
Use the compound interest formula and solve for t.

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$
$$10{,}000 = 2000\left(1 + \frac{.08}{2}\right)^{2t}$$
$$5 = (1.04)^{2t}$$
$$\ln(5) = \ln(1.04)^{2t}$$
$$\ln(5) = 2t\ln(1.04)$$
$$t = \frac{\ln(5)}{2\ln(1.04)}$$

14.A

To find the relative extrema, take the first derivative using the product rule and create a first derivative number line. Remember that e^x will never = 0 and will always be positive.

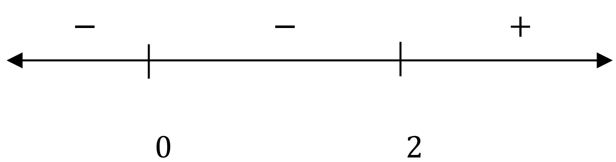
$$f'(x) = xe^{-x}(-1) + e^{-x}$$
$$f'(x) = e^{-x}(-x+1)$$
$$0 = e^{-x}(1-x)$$



15.C

Create a second derivative number line. Inflection points occur anywhere the concavity changes.

$$f'(x) = 15x^4 - 40x^3 + 8$$
$$f''(x) = 60x^3 - 120x^2$$
$$0 = 60x^2(x-2)$$
$$x = 0, x = 2$$



16. D

We are looking for the slope of the tangent line at the point $x = e$. We find the slope of a tangent line by taking the derivative of the function. Here, we will want to simplify the function before we take the derivative.

$$y = x^5 \ln\left(x^3\right)$$
$$y = \left(x^5\right) \cdot 3\ln x$$

Now that we've simplified the function, we can take the derivative by using the product rule.

$$y' = \left(x^5\right)(3)\left(\frac{1}{x}\right) + 3\ln x\left(5x^4\right)$$

Since we were given a point, plug the point in immediately after you find the derivative. Once you've plugged in for x, simpifly.

$$y'(e) = \left(e^5\right)(3)\left(\frac{1}{e}\right) + 3\ln e\left(5e^4\right)$$
$$y'(e) = \left(\frac{3e^5}{e}\right) + 3(1)\left(5e^4\right)$$
$$y'(e) = 3e^4 + 15e^4$$
$$y'(e) = 18e^4$$

17.A

Take the derivate using implicit differentiation.

$$(x+y)^{1/2} = 3x^2 - 7$$

$$\frac{1}{2}(x+y)^{-1/2}\left(1+\frac{dy}{dx}\right) = 6x$$

$$\frac{1}{2\sqrt{x+y}}\left(1+\frac{dy}{dx}\right) = 6x$$

$$\left(1+\frac{dy}{dx}\right) = 6x\left(\frac{2\sqrt{x+y}}{1}\right)$$

$$1+\frac{dy}{dx} = 12x\sqrt{x+y}$$

$$\frac{dy}{dx} = 12x\sqrt{x+y} - 1$$

18. B

The first thing to do is solve for x in order to find $f(p)$, and $f'(p)$. Then plug into the elasticity equation. You also need to know that revenue is maximized when $E = 1$

$$2p + \frac{x}{50} = 8$$

$$\frac{x}{50} = 8 - 2p$$

$$x = 400 - 100p$$

$$f(p) = 400 - 100p$$

$$f'(p) = -100$$

$$E = \frac{-p \times f'(p)}{f(p)}$$

$$1 = \frac{(-p)(-100)}{400 - 100p}$$

$$400 - 100p = 100p$$

$$400 = 200p$$

$$p = 2$$

19. E

This is a related rates problem. Start by finding the revenue function, then take the derivative with respect to time.

$$R = px$$
$$R = \left(250 - \frac{x}{20}\right)x$$
$$R = 250x - \frac{x^2}{20}$$
$$\frac{dR}{dt} = 250\frac{dx}{dt} - \frac{1}{10}x\frac{dx}{dt}$$
$$\frac{dR}{dt} = 250(10) - \frac{1}{10}(10)(500)$$
$$\frac{dR}{dt} = 2500 - 500 = 2000$$

20. E

Take the integral to find the Revenue function. Then use the point $R(0) = 0$ to solve for C. Then plug in 100 for x.

$$R(x) = 10\frac{e^{2x}}{2} + 2\frac{x^2}{2} + C$$
$$R(x) = 5e^{2x} + x^2 + C$$
$$0 = 5(e)^0 + (0) + C$$
$$0 = 5 + C$$
$$C = -5$$
$$R(x) = 5e^{2x} + x^2 - 5$$

21. B

We are given that:

$P'(x) = -2x + 20$

$P(0) = -40$

We want to know the maximum profit. Remember that profit is maximized so we will set $P'(x) = 0$ to find out what number we ultimately need to plug into our profit equation.

$-2x + 20 = 0$

$x = 10$

$P(x) = \int -2x + 20dx$

$P(x) = -x^2 + 20x + C$

Use $P(0) = -40$ to solve for C.

$-40 = -(0)^2 + 20(0) + C$

$C = -40$

$P(x) = -x^2 + 20x - 40$

$P(10) = -100 + 200 - 40 = 60$

22. B

Use substitution to integrate.

$u = 1 + e^{5x}$

$du = 5e^{5x}dx$

$dx = \frac{du}{5e^{5x}}$

$\int \frac{e^{5x}}{u} \frac{du}{5e^{5x}}$

$\int \frac{1}{5} \bullet \frac{1}{u} du$

$\frac{1}{5} \ln u$

$\frac{1}{5} \ln(e^{5x} + 1) + C$