LionTutors

**MATH110 – Exam 3 - Sample Test – Detailed Solutions**

1. **E**

   Take the derivative using the quotient rule and then set the top and bottom equal to 0.
   **Remember that critical numbers must be in the domain of $f(x)$**, so we can immediately cross off answer choices B and D because they contain $x = 2$.

$$f'(x) = \frac{(x-2)5e^{4x}(4) - 5e^{4x}(1)}{(x-2)^2}$$ *Quotient Rule*

$$f'(x) = \frac{5e^{4x}\left(4(x-2)-1\right)}{(x-2)^2}$$ *Factor out* $5e^x$

$$f'(x) = \frac{5e^{4x}\left(4x-9\right)}{(x-2)^2}$$ *Simplify*

| TOP | BOTTOM | |
|---|---|---|
| $5\cancel{e^{4x}}\left(4x-9\right)=0$ | $\left(x-2\right)^2=0$ | |
| $4x-9=0$ | $x-2=0$ | Set top and bottom equal to 0. |
| $x=9/4$ | $\cancel{x=2}$ | |

Remember that, since all exponential functions are positive, $e^x$ will never equal 0, and $x = 2$ is not in the domain. So $x = 9/4$ is the only critical number.

2. **A**
Since there is only a constant in the numerator (no variable), we can bring the entire denominator up to the top. When we do this, the exponent will change signs:

$$f(x)=2\left(4+3e^{-2x}\right)^{-5}$$

This allows us to use the chain rule to take the derivative:

$$f'(x)=-10\left(4+3e^{-2x}\right)^{-6}\left(3e^{-2x}\right)\left(-2\right)$$

Then, you can bring the negative exponent back down to the denominator and simplify what's left in the numerator:

$$f'(x)=\frac{60e^{-2x}}{\left(4+3e^{-2x}\right)^{6}}$$

3. **A**
Use the formula for compound interest:

$$A=P\left(1+\frac{r}{m}\right)^{mt}$$

$$A=2000\left(1+\frac{.10}{4}\right)^{(4)(5)}$$

$$A=2000(1.025)^{20}$$

**4. B**

To find the carrying capacity, factor out the bottom and simplify. Then carrying capacity is equal to the number on top.:

$$s(t)=\frac{9000}{3+6e^{-3t}}=\frac{9000}{3(1+2e^{-3t})}=\frac{3000}{1+2e^{-3t}}$$

Since we know our carrying capacity is 3,000, we can eliminate answer choices C and D.

To find $s(0)$, plug in 0 for $t$. Remember that $e^0=1$.

5. **A**

$$f(x)=\ln\frac{10x^4}{(4x^3-6)^2}$$

$$f(x)=\ln\left(10x^4\right)-\ln\left(4x^3-6\right)^2$$

$$f(x)=4\ln(10x)-2\ln\left(4x^3-6\right)$$

$$f'(x)=4\left(\frac{1}{10x}\right)\left(10\right)-2\left(\frac{1}{4x^3-6}\right)\left(12x^2\right)$$

$$f'(x)=\frac{40}{10x}-\frac{24x^2}{4x^3-6}$$

$$f'(x)=\frac{4}{x}-\frac{24x^2}{4x^3-6}$$

**6. E**

Start by finding the revenue equation, then simplify.

$$R(x) = px$$
$$R(x) = (x^2 e^{3x} + 1)x$$
$$R(x) = x^3 e^{3x} + x$$

Now, take the derivative.

$$R'(x) = \left(x^3\right)\left(e^{3x}\right)\left(3\right) + \left(e^{3x}\right)\left(3x^2\right) + 1$$

Now, since we know that the quantity supplied is 1,000 ($x = 1$), find $R'(1)$. Plug in a 1 everywhere that you have an $x$ right after you take the derivative, before simplifying.

$$R'(1) = \left(1^3\right)\left(e^{3(1)}\right)\left(3\right) + \left(e^{3(1)}\right)\left(3(1)^2\right) + 1$$
$$R'(1) = \left(1\right)\left(e^3\right)\left(3\right) + \left(e^3\right)\left(3\right) + 1$$
$$R'(1) = 3e^3 + 3e^3 + 1$$
$$R'(1) = 6e^3 + 1$$

**7. D**

We are looking for the slope of the tangent line at the point $x = e$. We find the slope of a tangent line by taking the derivative of the function. Here, we will want to simplify the function before we take the derivative.

$$y = x^5 \ln\left(x^3\right)$$
$$y = \left(x^5\right) \cdot 3\ln x$$

Now that we've simplified the function, we can take the derivative by using the product rule.

$$y' = \left(x^5\right)(3)\left(\frac{1}{x}\right) + 3\ln x\left(5x^4\right)$$

Since we were given a point, plug the point in immediately after you find the derivative. Once you've plugged in for $x$, simpifly.

$$y'(e) = \left(e^5\right)(3)\left(\frac{1}{e}\right) + 3\ln e\left(5e^4\right)$$
$$y'(e) = \left(\frac{3e^5}{e}\right) + 3(1)\left(5e^4\right)$$
$$y'(e) = 3e^4 + 15e^4$$
$$y'(e) = 18e^4$$

**8. E**

Use $A = Pe^{rt}$ and solve for r.

$$7000 = 5000e^{3r}$$
$$\frac{7}{5} = e^{3r}$$
$$\ln\left(\frac{7}{5}\right) = 3r$$
$$r = \frac{\ln\left(\frac{7}{5}\right)}{3}$$

9. **A**

Since we are looking for dy/dx we need to use implicit differentiation. We will be taking the derivative of everything **with respect to x**.

$$\ln(x+y) = x^3 + 41$$

$$\frac{1}{(x+y)}\left(1+\frac{dy}{dx}\right) = 3x^2$$ We are taking the derivative with respect to $x$, so we need to tack on a dy/dx when we differentiate the $y$ term

$$1+\frac{dy}{dx} = 3x^2(x+y)$$ Multiply both sides by (x+y)

$$\frac{dy}{dx} = 3x^2(x+y)-1$$ Subtract 1 from both sides to get dy/dx alone

**10. D**

Simplify within the integral before integrating. Remember that $\sqrt{t} = t^{1/2}$. Then, use the rules of integration to take the antiderivative and simplify.

$$\int \sqrt{t}\left(t-1\right)dt$$
$$\int t^{3/2} - t^{1/2}dt$$
$$= \frac{t^{5/2}}{5/2} - \frac{t^{3/2}}{3/2} + C$$
$$= \frac{2}{5}t^{5/2} - \frac{2}{3}t^{3/2} + C$$
$$= 2t^{3/2}\left(\frac{1}{5}t - \frac{1}{3}\right) + C$$

**11. B**

Take the integral, then use the fixed costs to solve for $C$. If fixed costs are \$400, then that means C(0)=\$400.

$$C(x) = \int\left(-0.4x+100\right)dx$$
$$C(x) = -0.4\frac{x^2}{2} + 100x + C$$
$$C(x) = -0.2x^2 + 100x + C$$
$$400 = 0 + 0 + C$$
$$C = 400$$
$$C(x) = -0.2x^2 + 100x + 400$$
$$C(100) = -0.2(100)^2 + 100(100) + 400$$
$$C(100) = -0.2(10{,}000) + 10{,}000 + 400$$
$$C(100) = -2000 + 10{,}000 + 400$$
$$C(100) = 8400$$

**12. B**

Start by taking the integral, using a u-substitution, then use the information given to solve for C.

$$p(x) = \int \frac{-500x}{(9+x^2)^{3/2}} dx$$

$$p(x) = \int \frac{-500x}{(u)^{3/2}} \frac{du}{2x}$$

$$p(x) = \int -250u^{-3/2} du$$

$$p(x) = -250 \frac{u^{-1/2}}{-1/2} + C$$

$$p(x) = \frac{500}{\sqrt{u}} + C$$

$$p(x) = \frac{500}{\sqrt{9+x^2}} + C$$

$$130 = \frac{500}{\sqrt{9+(4)^2}} + C$$

$$130 = 100 + C$$

$$C = 30$$

$$p(x) = \frac{500}{\sqrt{9+x^2}} + 30$$

$$u = 9 + x^2$$

$$du = 2x dx$$

$$dx = \frac{du}{2x}$$

**13. E**

Take the integral to find the Revenue function. Then use the point $R(0) = 0$ to solve for $C$. Then plug in 100 for $x$.

$$R(x) = 10\frac{e^{2x}}{2} + 2\frac{x^2}{2} + C$$
$$R(x) = 5e^{2x} + x^2 + C$$
$$0 = 5(e)^0 + (0) + C$$
$$0 = 5 + C$$
$$C = -5$$
$$R(x) = 5e^{2x} + x^2 - 5$$

**14. B**

We are given that:

$P'(x) = -2x + 20$

$P(0) = -40$

We want to know the maximum profit. Remember that profit is maximized so we will set $P'(x) = 0$ to find out what number we ultimately need to plug into our profit equation.

$-2x + 20 = 0$

$x = 10$

$P(x) = \int -2x + 20dx$

$P(x) = -x^2 + 20x + C$

Use $P(0) = -40$ to solve for C.

$-40 = -(0)^2 + 20(0) + C$

$C = -40$

$P(x) = -x^2 + 20x - 40$

$P(10) = -100 + 200 - 40 = 60$

**15. B**

Use substitution to integrate.

$u = 1 + e^{5x}$

$du = 5e^{5x}dx$

$dx = \frac{du}{5e^{5x}}$

$\int \frac{e^{5x}}{u} \frac{du}{5e^{5x}}$

$\int \frac{1}{5} \bullet \frac{1}{u} du$

$\frac{1}{5} \ln u$

$\frac{1}{5} \ln(e^{5x} + 1) + C$

**16. C**

Use substitution to integrate.

$u = \ln x$

$du = \frac{1}{x} dx$

$dx = x du$

$\int \frac{u^2}{x} x \, du$

$\int u^2 \, du$

$\frac{u^3}{3} = \left. \frac{(\ln x)^3}{3} \right|_1^e$

$\frac{(\ln e)^3}{3} - \left( \frac{(\ln 1)^3}{3} \right) = \frac{1}{3} - 0$

**17. B**

You will need to use substitution, then use "backdoor substitution" to go back and solve for $x$ in terms of $u$.

$$u = x - 2$$
$$du = dx$$
$$x = u + 2$$
$$\int \frac{x}{u} du$$
$$\int \frac{u+2}{u} du$$
$$\int \frac{u}{u} + \frac{2}{u} du$$
$$\int 1 + 2u^{-1}\, du$$
$$u + 2\ln u + C$$
$$x - 2 + 2\ln(x-2) + C$$
$$x + 2\ln(x-2) + C$$

**18. C**

Start by taking the second derivative and setting it equal to 0. **Remember that you can never take the log of a negative number or 0!**

$$f'(x) = \frac{72}{x} + 2x$$
$$f'(x) = 72x^{-1} + 2x$$

$$f''(x) = -\frac{72}{x^2} + 2$$

$$0 = -\frac{72}{x^2} + 2$$
$$\frac{72}{x^2} = 2$$
$$2x^2 = 72$$
$$x^2 = 36$$
$$x = -6, x = 6$$

Since you cannot take the log of a negative number, $x = 6$ is the only potential inflection point. Remember, **inflection points must be in the domain of the function.** Start by taking the second derivative and setting it equal to 0. **Remember that you can never take the log of a negative number or 0!**

We just need to make sure the concavity changes there by putting it on a second derivative number line.


**19. C**

This statement is not true. You cannot expand when you are adding or subtracting inside of a log.

**20.A**

$$\ln\left(x-y\right)=x^2+7$$

$$\frac{1}{\left(x-y\right)}\left(1-\frac{dy}{dx}\right)=2x$$

$$\left(1-\frac{dy}{dx}\right)=2x\left(x-y\right)$$

$$-\frac{dy}{dx}=2x\left(x-y\right)-1$$

$$\frac{dy}{dx}=1-2x\left(x-y\right)$$