MATH110 – Final Exam (New Material) – Sample Test Solutions

1. **B**
   Use u-substitution to take the integral. Let $u = x^3$.

$$\int 3x^2 e^u \frac{du}{3x^2}$$

$$\int e^u \, du$$

$$e^u$$

$$e^{x^3}\Big|_0^1 = e^1 - e^0 = e - 1$$

$$u = x^3$$

$$du = 3x^2 dx$$

$$dx = \frac{du}{3x^2}$$

2. **E**
   Use the average value formula with $b = 3,\ a = 0$. You must use u-sub to integrate.

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\frac{1}{3-0}\int_0^3 te^{t^2}\,dt$$

$$\frac{1}{3}\int_0^3 te^u \frac{du}{2t}$$

$$\frac{1}{3}\int_0^3 \frac{1}{2}e^u\,du$$

$$\frac{1}{6}\int_0^3 e^u\,du$$

$$\frac{1}{6}e^u = \frac{1}{6}e^{t^2}\Big|_0^3$$

$$\frac{1}{6}\left(e^{3^2} - e^{0^2}\right) = \frac{1}{6}\left(e^9 - 1\right)$$

3. **A**

This is a producer surplus question. Start by setting the equation equal to each other to obtain $x$and p.

$$70 - 0.1x = 20 + 0.4x$$
$$50 = 0.5$$
$$x = 100$$
$$p = 70 - .1(100) = 60$$
$$PS = (100)(60) - \int_0^{100} \left(S(x)\right)dx$$
$$= 6000 - \int_0^{100} \left(20 + .4x\right)dx$$
$$= 6000 - \left[20x + .2x^2\right]\Big|_0^{100}$$
$$= 6000 - [(20)(100) + .2(100)^2 - 0]$$
$$= 6000 - [2000 + 2000]$$
$$= 2000$$

4. **C**

This is an area between two curves problem. First, find the intersection points of the given function by setting them equal to one another. Then, sketch the functions to determine which function is the top curve and which is the bottom curve.

$$x^4 = 1$$
$$x = 1, -1$$

Then, find the area between the curves by calculating the following integral:

$$\int_a^b (\boldsymbol{Top\ Curve} - \boldsymbol{Bottom\ Curve})\ \boldsymbol{dx}$$

$$\int_{-1}^{1} (1 - x^4)dx = x - \frac{x^5}{5}\Big|_{-1}^{1}$$

$$= \left[1 - \frac{1}{5}\right] - \left[-1 - \frac{-1}{5}\right] = 1 - \frac{1}{5} + 1 - \frac{1}{5} = 2 - \frac{2}{5} = 8/5$$

5. **D**
Use the FV of an Income Stream formula.

$$e^{.10(20)}\int_{0}^{20} 8000e^{-.1t}\,dt$$

$$= e^{2}\int_{0}^{20} 8000e^{-.1t}\,dt$$

6. **A**
Use the formula for the amount of an annuity.

$$A = \frac{2400}{.07}(e^{0.7} - 1)$$

7. **E**
This is an area between two curves problem. First, find the intersection points of the given function by setting them equal to one another.

$$x^2 = \sqrt{x}$$
$$x^4 = x$$
$$x = 0,1$$

Next, you know that $\sqrt{x}$ is the top curve. You can figure this out visually by graphing, or you can plug in a number between 0 and 1 into both functions to see which function gives you the bigger value.

Now we integrate by calculating the following integral:

$$\int_{a}^{b} (\boldsymbol{Top\ Curve} - \boldsymbol{Bottom\ Curve})\ \boldsymbol{dx}$$

$$\int_{0}^{1} x^{1/2} - x^2 dx$$

$$= \frac{2}{3}x^{3/2} - \frac{1}{3}x^3\Big|_0^1$$

$$= \left(\frac{2}{3} - \frac{1}{3}\right) - (0 - 0) = \frac{1}{3}$$

8. **A**

Use the average value formula. You will have to use substitution to integrate.

$$\frac{1}{3-1}\int_1^3 \frac{2t}{t^2+1}dt$$

$$\frac{1}{2}\int_1^3 \frac{2t}{u}\frac{du}{2t}$$

$$\frac{1}{2}\int_1^3 u^{-1}\,du$$

$$\frac{1}{2}\ln u = \frac{1}{2}\ln(t^2+1)\Big|_1^3$$

$$\frac{1}{2}\ln(3^2+1) - \frac{1}{2}\ln(1^2+1)$$

$$\frac{1}{2}\ln 10 - \frac{1}{2}\ln 2 = \frac{1}{2}\ln 5$$

9. **C**

Use substitution to integrate.

$$u = \ln x$$

$$du = \frac{1}{x}dx$$

$$dx = xdu$$

$$\int \frac{u^2}{x} x\,du$$

$$\int u^2\,du$$

$$\frac{u^3}{3} = \frac{(\ln x)^3}{3}\Big|_1^e$$

$$\frac{(\ln e)^3}{3} - \left(\frac{(\ln 1)^3}{3}\right) = \frac{1}{3} - 0$$

10. **C**

Use u-substitution to integrate. Then evaluate from 0 to 2.

$$u = 1 + 2x^2$$
$$du = 4x dx$$
$$dx = \frac{du}{4x}$$

$$\int \frac{8x}{\sqrt{u}} \frac{du}{4x}$$
$$\int 2u^{-1/2}\, du$$
$$2\frac{u^{1/2}}{1/2} = 4\sqrt{u} = 4\sqrt{1+2x^2}\Big|_0^2$$
$$= \left(4\sqrt{1+2(2)^2}\right) - \left(4\sqrt{1+2(0)^2}\right)$$
$$= 4\sqrt{9} - 4\sqrt{1} = 12 - 4 = 8$$

11. **D**

Use the formula for Value of an Income Stream.

$$e^{.05(8)} \int_0^8 2000e^{-.05t}\, dt$$

$$= e^{.4} \int_0^8 2000e^{-.05t}\, dt$$

12. **A**

Use the formula for PV of an Income Stream

$$\int_0^T R(t)e^{-rt}\, dt$$

$$\int_0^{12} 6000e^{-.05t}\, dt$$

13. **B**

This is a consumer surplus question. Start by finding the equilibrium quantity and price and then use the formula for Consumer Surplus.

$$20-\frac{1}{3}x=12+x$$

$$8=\frac{4}{3}x$$

$$x=6$$
$$p=12+6=18$$

$$CS=\int_0^6\left(20-\frac{1}{3}x\right)dx-108$$

$$CS=\left[20x-\frac{1}{6}x^2\right]_0^6-108$$

$$CS=\left[\left(120-6\right)-(0)\right]-108$$

$$CS=114-108=6$$

14. **C**

Use the formula for PV of an annuity.

$$PV=\frac{(12)(1{,}000)}{0.09}\left(1-e^{-(0.09)(8)}\right)$$

$$PV=\frac{12{,}000}{0.09}(1-e^{-0.72})$$

15. **B**

When a question asks "find the area of the region under the graph" that just means to integrate. There are two methods for integrating this function. You can use integration by parts with $u = \ln x$, $dv = dx$. Or you can just memorize that $\int \ln x \, dx = x \ln x - x$

$$\int_1^4 \ln x \, dx = x \ln x - x \Big|_1^4$$
$$= \left[4\ln(4) - 4\right] - \left[1\ln(1) - 1\right]$$
$$= 4\ln(4) - 4 - 0 + 1$$
$$= 4\ln(4) - 3$$

16. **B**

Since normal u-substitution does not work, you have to use integration by parts.

$$\int xe^{2x} \, dx = uv - \int v \, du$$
$$2x(e^x) - \int e^x 2 \, dx \qquad u = 2x \qquad v = e^x$$
$$2xe^x - 2\int e^x \, dx \qquad du = 2dx \qquad dv = e^x dx$$
$$2xe^x - 2e^x + C$$

17. **D**

Since normal u-substitution does not work, you have to use integration by parts.

$$\int x \ln x = uv - \int v \, du$$
$$\ln x\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2}\frac{1}{x} dx \qquad u = \ln x \qquad v = \frac{x^2}{2}$$
$$\frac{1}{2}x^2 \ln x - \frac{1}{2}\int x \, dx \qquad du = \frac{1}{x}dx \qquad dv = xdx$$
$$\frac{1}{2}x^2 \ln x - \frac{1}{2}\left(\frac{x^2}{2}\right) + C$$