Problem 1

a) order = 2

b) linear

c) $\left.\begin{array}{l} y = c \\ y' = 0 \\ y'' = 0 \end{array}\right\} \rightarrow$ $3C - 0 + 0 = 1$

$C = \frac{1}{3}$

$$\boxed{y = \frac{1}{3}}$$

Problem 2

a) True

b) True

c) False → If $y = 0, y' = 0$

$yy' = 10$

$(0)(0) = 10$

$0 = 10$ ← NOT TRUE

d) True → If $y = 1, y' = 0$

$0 - t^2 \sin(\pi) = 0$

$0 = 0$ ← TRUE

Problem 3

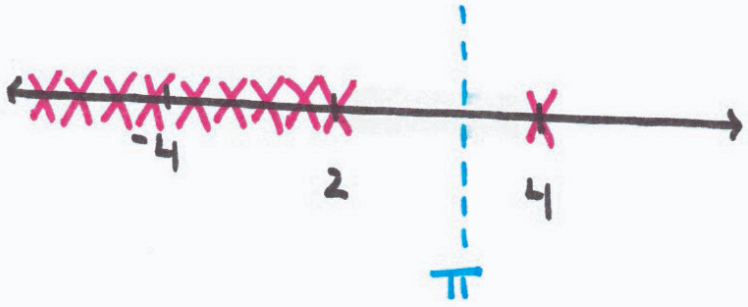
Put the equation in standard form...

$$y' + \frac{\ln(t-2)}{t^2-16}\,y = \frac{\cos(2t)}{t^2-16}$$

$\ln(t-2)$
$t-2>0$
$t>2$

$t^2-16\neq 0$
$t\neq \pm 4$

$t\neq \pm 4$



$y(\pi)=5$

Interval: $(2,4)$

Problem 4

Use the Integrating Factor Method.

$$y' + \frac{1}{2\sqrt{t}}\, y = 1$$

$p(t) = \frac{1}{2\sqrt{t}}$ $\qquad$ $g(t) = 1$

$$\mu(t) = e^{\int p(t)\,dt} = e^{\int \frac{1}{2} t^{-1/2}\,dt} = e^{\sqrt{t}}$$

$$y = \frac{1}{e^{\sqrt{t}}}\left[\int e^{\sqrt{t}} \cdot 1 \, dt\right]$$

To evaluate $\int e^{\sqrt{t}}\,dt$ use substitution and I.B.P.

$$\int e^{\sqrt{t}}\,dt \rightarrow \begin{cases} w = \sqrt{t} \\ dw = \frac{1}{2\sqrt{t}}\,dt \\ dt = 2\sqrt{t}\,dw \end{cases}$$

$$= \int e^{w} 2\sqrt{t}\,dw$$

$$= \int 2we^{w}\,dw \rightarrow \begin{cases} u = 2w \\ du = 2\,dw \end{cases} \qquad \begin{matrix} v = e^{w} \\ dv = e^{w}\,dw \end{matrix}$$

$$= 2we^{w} - \int 2e^{w}\,dw$$

$$= 2we^{w} - 2e^{w} + C$$

$$= 2\sqrt{t}e^{\sqrt{t}} - 2e^{\sqrt{t}} + C$$

$$y = \frac{1}{e^{\sqrt{t}}}\left[2\sqrt{t}e^{\sqrt{t}} - 2e^{\sqrt{t}} + C\right]$$

$$y = 2\sqrt{t} - 2 + \frac{c}{e^{\sqrt{t}}}$$

$$y(1) = -2 \rightarrow -2 = 2 - 2 + \frac{c}{e}$$

$$-2 = \frac{c}{e}$$

$$c = -2e$$

$$\boxed{y = 2\sqrt{t} - 2 - \frac{2e}{e^{\sqrt{t}}}}$$

Problem 5

$$y' + 2ty = e^{-t^2}\cos t + t^3$$

p $\qquad$ g

$$u(t) = e^{\int 2t\,dt} = e^{t^2}$$

$$y = \frac{1}{e^{t^2}}\left[\int e^{t^2}\left(e^{-t^2}\cos t + t^3\right)dt\right]$$

$$y = \frac{1}{e^{t^2}}\left[\int \cos t\,dt + \int t^3 e^{t^2}\,dt\right]$$

$$\int \cos(t)\,dt = \sin t$$

$$\int t^3 e^{t^2}\,dt \rightarrow$$

Substitution

$w = t^2$

$dw = 2t\,dt$

$dt = \frac{dw}{2t}$

$$\int t^3 e^{w}\,\frac{dw}{2t}$$

$$\int \frac{1}{2} t^2 e^{w}\,dw$$

$$\frac{1}{2}\int we^{w} \rightarrow$$

I.B.P.

$u = w$ $\qquad$ $v = e^w$

$du = dw$ $\qquad$ $dv = e^w\,dw$

$$\frac{1}{2}\left[we^{w} - \int e^{w}\,dw\right]$$

$$\frac{1}{2}\left[t^2 e^{t^2} - e^{t^2}\right] + C$$

$$y = \frac{1}{e^{t^2}}\left[\sin t + \frac{1}{2}t^2 e^{t^2} - \frac{1}{2}e^{t^2} + C\right]$$

$$\boxed{y = \frac{\sin t}{e^{t^2}} + \frac{1}{2}t^2 - \frac{1}{2} + \frac{C}{e^{t^2}}}$$

Problem 6

This is a separable ODE...

$$\frac{dy}{dx} = \frac{2\cos(2x)}{y+1}$$

$$\int (y+1)\,dy = \int 2\cos(2x)\,dx$$

$$\frac{1}{2}y^2 + y = \sin(2x) + C$$

$$y^2 + 2y = 2\sin(2x) + C$$

$y(0) = 3 \rightarrow \quad (3)^2 + 2(3) = 2\sin(0) + C$

$$15 = C$$

$$y^2 + 2y \boxed{+1} = 2\sin(2x) + 15 \boxed{+1}$$

$$(y+1)^2 = 2\sin(2x) + 16$$

$$y+1 = \pm\sqrt{2\sin(2x) + 16}$$

$y = -1 \pm \sqrt{2\sin(2x) + 16}$, but since $y(0) = 3$ we only want the positive root

$$\boxed{y = -1 + \sqrt{2\sin(2x) + 16}}$$

Problem 7

a) $\left.\begin{matrix} y = k \\ y' = 0 \end{matrix}\right\} \rightarrow$

$0 = k^2 - 2k$

$0 = k(k-2)$

$k = 0, \; k = 2$

constant sol. $y = 0, \; y = 2$

b) $\frac{dy}{dt} = y^2 - 2y$

$\frac{1}{y^2 - 2y} \, dy = 1 \, dt$

$\int \frac{1}{y(y-2)} \, dy = \int 1 \, dt$

P.F.D.

$\frac{1}{y(y-2)} = \frac{A}{y} + \frac{B}{y-2}$

$A = -\frac{1}{2}, \quad B = \frac{1}{2}$

$\int \left(\frac{1/2}{y-2} - \frac{1/2}{y} \right) dy = \int 1 \, dt$

$\frac{1}{2} \left[\ln|(y-2)| - \ln|y| \right] = t + C$

$\ln \sqrt{\frac{y-2}{y}} = t + C$

$\sqrt{\frac{y-2}{y}} = e^{t+C}$

$\sqrt{\frac{y-2}{y}} = ce^{t}$

$\frac{y-2}{y} = (ce^{t})^2$

$\frac{y-2}{y} = ce^{2t}$

$y - 2 = yce^{2t}$

$y - yce^{2t} = 2$

$y(1 - ce^{2t}) = 2$

$$y = \frac{2}{1 - ce^{2t}}$$

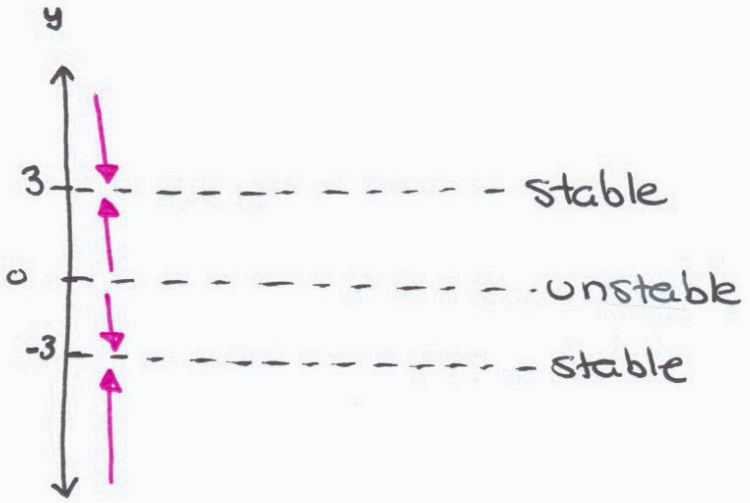
Problem 8

a) $0 = 9y - y^3$

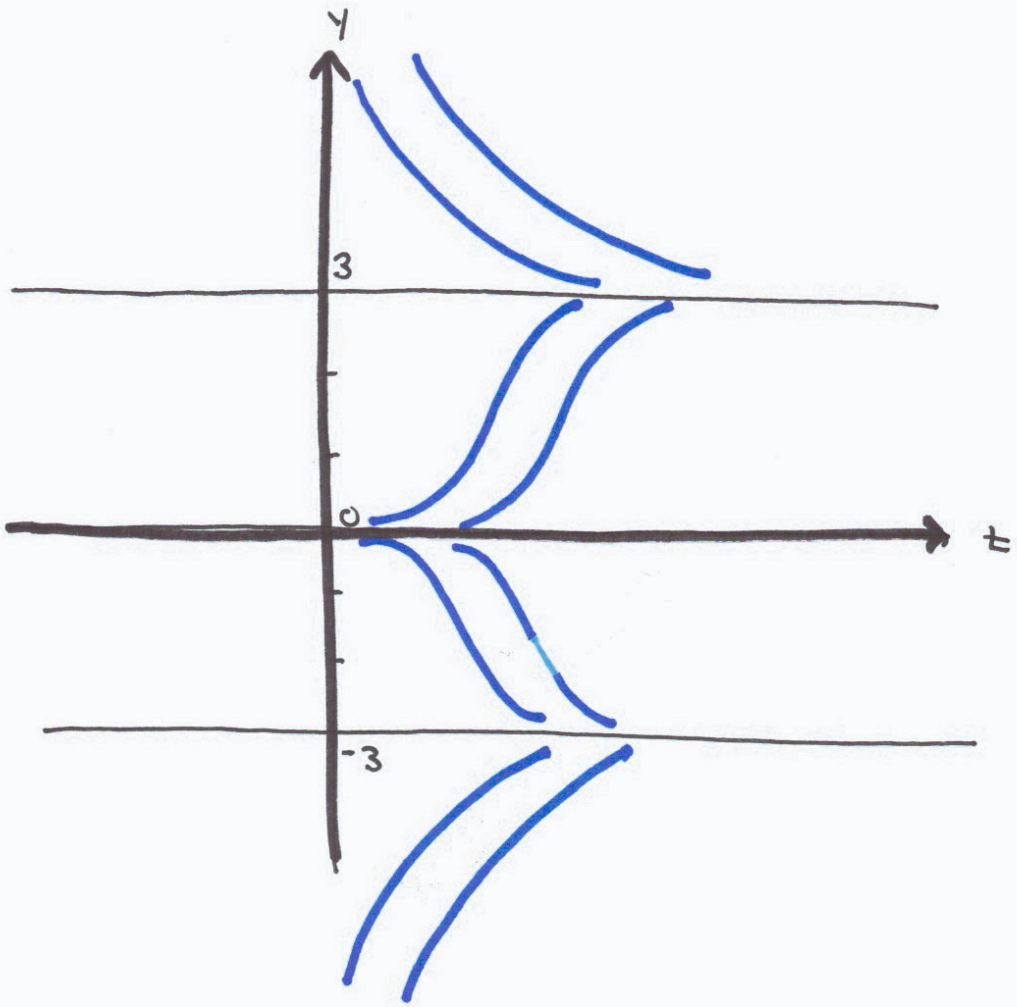
$0 = y(9 - y^2)$

$0 = y(3 - y)(3 + y)$

$y = 0,\ y = 3,\ y = -3$



b)



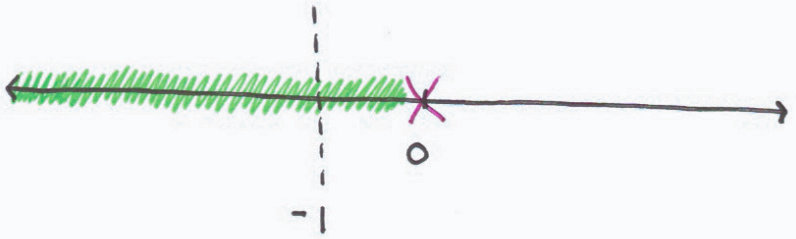
Problem 9

a) True

b) True by "Principle of Superpositron"

c) $y'' + \frac{4}{t^2} y' + \frac{3}{t^2} y = 0$

$t \neq 0$ $\quad$ $t \neq 0$



$(-\infty, 0)$

Problem 10

a) $r^2 + 3 = 0$

$r^2 = -3$

$r = 0 \pm \sqrt{3}i$

$y = c_1 e^{0t} \cos(\sqrt{3}t) + c_2 e^{0t} \sin(\sqrt{3}t)$

$$y = c_1 \cos(\sqrt{3}t) + c_2 \sin(\sqrt{3}t)$$

b) $r^2 + r - 2 = 0$

$(r + 2)(r - 1) = 0$

$r = -2, \; r = 1$

$$y = c_1 e^{-2t} + c_2 e^{t}$$

c) $r^2 + 6r + 9 = 0$

$(r + 3)(r + 3) = 0$

$r = -3, \; r = -3$

$$y = c_1 e^{-3t} + c_2 t e^{-3t}$$

Problem 11

$r^2 + 4r + 4 = 0$

$(r+2)(r+2) = 0$

$r = -2, r = -2 \rightarrow$ Repeated

$$y = c_1 e^{-2t} + c_2 t e^{-2t}$$

$y(0) = -6$

$-6 = c_1$

$$y = -6e^{-2t} + c_2 t e^{-2t}$$

$$y' = 12e^{-2t} - 2c_2 t e^{-2t} + c_2 e^{-2t}$$

$y'(0) = 8$

$8 = 12 + c_2$

$c_2 = -4$

$$\boxed{y = -6e^{-2t} - 4te^{-2t}}$$

Problem 12

a) Find $y_c(t)$:

$r^2 + 3r = 0$

$r(r+3) = 0$

$r = 0, \quad r = -3$

$y_c(t) = c_1 + c_2 e^{-3t}$

Find $y_p(t)$:

$y_{p1}: y'' + 3y' = 5e^{3t}$

$y_{p1} = Ae^{3t}$

$y_{p1}' = 3Ae^{3t}$

$y_{p1}'' = 9Ae^{3t}$

$9Ae^{3t} + 3(3Ae^{3t}) = 5e^{3t}$

$18Ae^{3t} = 5e^{3t}$

$A = \frac{5}{18}$

$y_{p1} = \frac{5}{8}e^{3t}$

$y_{p2}: y'' + 3y' = 7$

$y_{p2} = B \rightarrow$ overlaps y_c, so modify

$y_{p2} = Bt$

$y_{p2}' = B$

$y_{p2}'' = 0$

$0 + 3B = 7$

$B = \frac{7}{3}$

$y_{p2} = \frac{7}{3}t$

$$y_p(t) = \frac{5}{8}e^{3t} + \frac{7}{3}t$$

b) $$y = c_1 + c_2 e^{-3t} + \frac{5}{8}e^{3t} + \frac{7}{3}t$$

Problem 13

a) $r^2 + 4 = 0$

$r^2 = -4$

$r = \pm 2i$

$r = 0 \pm 2i$

e^{0t}

$r = 0 \pm 2i$ matches root of comp. sol. so extra power = 1

$$y_p = t^{1}(At^2 + Bt + C)\cos(2t) + t^{1}(Dt^2 + Et + F)\sin(2t)$$

$$\boxed{y_p = t(At^2 + Bt + C)\cos(2t) + t(Dt^2 + Et + F)\sin(2t)}$$

b) $r^2 + 2r + 5 = 0$

$$r = \frac{-2 \pm \sqrt{4 - 20}}{2}$$

$$r = \frac{-2 \pm 4i}{2}$$

$r = -1 \pm 2i$

$y_{p1} = A\cos(3t) + B\sin(3t)$

$r = 0 \pm 3i$ no overlap

$y_{p2} = t^{1}(ct + D)e^{-t}\cos(2t) + t^{1}(Et + F)e^{-t}\sin(2t)$

$r = -1 \pm 2i$ matches, so extra power = 1

$$\boxed{\begin{aligned} y_p = {} & A\cos(3t) + B\sin(3t) + \\ & t(Ct + D)e^{-t}\cos(2t) + \\ & t(Et + F)e^{-t}\sin(2t) \end{aligned}}$$

c) $r^2 - 9 = 0$

$r^2 = 9$

$r = 3, r = -3$

$y_{p1} = t^{1}(At+B)e^{-3t}$ → $r=-3$ matches one root, Extra power = 1

$y_{p1} = t(At+B)e^{-3t}$

$y_{p2} = t^{0}(Ct+D)\cos(3t) + t^{0}(Et+F)\sin(3t)$ → $r = 0 \pm 3i$ no match, Extra power $r = 0$

$y_{p2} = (Ct+D)\cos(3t) + (Et+F)\sin(3t)$

$$y_p = t(At+B)e^{-3t} + (Ct+D)\cos(3t) + (Et+F)\sin(3t)$$

Problem 14

a) **False** by the General Principle of Superposition For Non-Homogenous ODE's.

y_1 is a solution to $y'' + 5y' + 4y = \sin(6t)$

y_2 is a solution to $y'' + 5y' + 4y = \sin(6t)$

So $2y_1 + 4y_2$ is a solution to...

$$y'' + 5y' + 4y = 2\sin(6t) + 4\sin(6t)$$

$$y'' + 5y' + 4y = \underline{6\sin(6t)}$$

↳ NOT $\sin(6t)$

b) **True**

$2y_1 - y_2$ is a solution to...

$$y'' + 5y' + 4y = 2\sin(6t) - \sin(6t)$$

$$y'' + 5y' + 4y = \sin(6t) \checkmark$$

Problem 15

Manipulate the integral to get in the form $\int_0^\infty e^{-st} f(t)\, dt$, then take the Laplace transform of $f(t)$.

$$\int_0^\infty e^{-st} e^{-4t} \cos(7t)\, dt$$

$$f(t) = e^{-4t} \cos(7t)$$

$$\mathcal{L}\{e^{-4t} \cos(7t)\} = \frac{s+4}{(s+4)^2 + 49}$$

Problem 16

$$1 + u_1(t)(t^2 - 1) + u_{2\pi}(t)(\cos t - t^2)$$

$$\mathcal{L}\{1\} + \mathcal{L}\{u_1(t)(t^2 - 1)\} + \mathcal{L}\{u_{2\pi}(t)(\cos t - t^2)\}$$

$$\frac{1}{s} + e^{-s}\, \mathcal{L}\{(t+1)^2 - 1\} + e^{-2\pi s} \mathcal{L}\{\cos(t + 2\pi) - (t + 2\pi)^2\}$$

$$\frac{1}{s} + e^{-s}\, \mathcal{L}\{t^2 + 2t + 1 - 1\} + e^{-2\pi s} \mathcal{L}\{\cos(t) - (t^2 + 4\pi t + 4\pi^2)\}$$

$$\frac{1}{s} + e^{-s}\, \mathcal{L}\{t^2 + 2t\} + e^{-2\pi s} \mathcal{L}\{\cos(t) - t^2 - 4\pi t - 4\pi^2\}$$

$$\frac{1}{s} + e^{-s} \left(\frac{2}{s^3} + \frac{2}{s^2}\right) + e^{-2\pi s} \left(\frac{s}{s^2 + 1} - \frac{2}{s^3} - \frac{4\pi}{s^2} - \frac{4\pi^2}{s}\right)$$

Problem 17

a. Use Partial Fractions, then take the inverse Laplace.

$$\frac{36-s}{s(s+6)^2}=\frac{A}{s}+\frac{B}{(s+6)}+\frac{C}{(s+6)^2}$$

$$36-s=A(s+6)^2+Bs(s+6)+Cs$$

$$s=0\rightarrow A=1$$
$$s=-6\rightarrow C=-7$$

$$36-s=1(s+6)^2+Bs(s+6)-7s$$

$$36-s=s^2+12s+36+Bs^2+6Bs-7s$$

Equating the coefficients gives us $1+B=0$, so $B=-1$

$$\frac{36-s}{s(s+6)^2}=\frac{1}{s}-\frac{1}{(s+6)}-\frac{7}{(s+6)^2}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}-\frac{1}{(s+6)}-\frac{7}{(s+6)^2}\right\}=\mathbf{1}-\boldsymbol{e^{-6t}}-\mathbf{7}\boldsymbol{e^{-6t}t}$$

b. Since the denominator does not factor, start by completing the square.

$$\frac{2s-2}{s^2+2s+\mathbf{1}+17-\mathbf{1}}$$

$$2\frac{s-1}{(s+1)^2+16}$$

$$2\frac{s+\mathbf{1}-1-\mathbf{1}}{(s+1)^2+16}$$

$$2\left[\frac{s+1}{(s+1)^2+16}-\frac{2}{(s+1)^2+16}\right]$$

Now take the inverse Laplace

$$2\mathcal{L}^{-1}\left[\frac{s+1}{(s+1)^2+16}-\frac{1}{\mathbf{2}}\frac{2*\mathbf{2}}{(s+1)^2+16}\right]$$

$$=2\left[e^{-t}\cos(4t)-\frac{1}{2}e^{-t}\sin(4t)\right]$$

c. Take inverse Laplace of each term separately. You will have to complete the square for the first term…

$$\mathcal{L}^{-1}\left\{e^{-3s}\frac{s}{s^2+6s+\mathbf{9}+25-\mathbf{9}}-5e^{-4s}\right\}$$

$$\mathcal{L}^{-1}\left\{e^{-3s}\frac{s}{(s+3)^2+16}-5e^{-4s}\right\}$$

$$\mathcal{L}^{-1}\left\{e^{-3s}\frac{s+\mathbf{3}-\mathbf{3}}{(s+3)^2+16}-5e^{-4s}\right\}$$

$$\mathcal{L}^{-1}\left\{e^{-3s}\left(\frac{s+3}{(s+3)^2+16}-\frac{3}{(s+3)^2+16}\right)-5e^{-4s}\right\}$$

$$\mathcal{L}^{-1}\left\{e^{-3s}\left(\frac{s+3}{(s+3)^2+16}-\frac{3}{(s+3)^2+16}\right)\right\}-\mathcal{L}^{-1}\{5e^{-4s}\}$$

$$= u_3(t)f(t-3)-5\delta(t-4)$$

Where $f(t)=\mathcal{L}^{-1}\left\{\left(\frac{s+3}{(s+3)^2+16}-\frac{\mathbf{1}}{\mathbf{4}}\frac{3(\mathbf{4})}{(s+3)^2+16}\right)\right\}$

$$f(t)=e^{-3t}\cos(4t)-\frac{3}{4}e^{-3t}\sin(4t)$$

$$f(t-3)=e^{-3(t-3)}\cos\big(4(t-3)\big)-\frac{3}{4}e^{-3(t-3)}\sin\big(4(t-3)\big)$$

Overall answer: $u_3(t)\left[e^{-3(t-3)}\cos\big(4(t-3)\big)-\frac{3}{4}e^{-3(t-3)}\sin\big(4(t-3)\big)\right]-5\delta(t-4)$

Problem 18

Start by finding the Laplace transform of both sides.

$$\mathcal{L}\{y''\} + 4\mathcal{L}\{y'\} + 3\mathcal{L}\{y\} = \mathcal{L}\{2\delta(t-1)\}$$

$$s^2Y - s \cdot y(0) - y'(0) + 4[sY - y(0)] + 3Y = 2e^{-s}$$

$$s^2Y - 2 + 4sY + 3Y = 2e^{-s}$$

$$s^2Y + 4sY + 3Y = 2e^{-s} + 2$$

$$(s^2 + 4s + 3)Y = 2e^{-s} + 2$$

$$Y = e^{-s}\frac{2}{(s+3)(s+1)} + \frac{2}{(s+3)(s+1)}$$

$$y = \mathcal{L}^{-1}\left\{e^{-s}\frac{2}{(s+3)(s+1)} + \frac{2}{(s+3)(s+1)}\right\}$$

Partial Fraction Decomposition:

$$\frac{2}{(s+3)(s+1)} = \frac{A}{s+3} + \frac{B}{s+1} \qquad \rightarrow A = -1, B = 1$$

Back to the equation:

$$y = \mathcal{L}^{-1}\left\{e^{-s}\left[\frac{-1}{s+3} + \frac{1}{s+1}\right]\right\} + \mathcal{L}^{-1}\left\{\frac{-1}{s+3} + \frac{1}{s+1}\right\}$$

$$y = u_1(t)f(t-1) - e^{-3t} + e^{-t}$$

where $f(t) = \mathcal{L}^{-1}\left\{\frac{-1}{s+3} + \frac{1}{s+1}\right\}$

$$f(t) = -e^{-3t} + e^{-t}$$

$$f(t-1) = e^{-3(t-1)} + e^{-(t-1)}$$

$$y = u_1(t)\left(e^{-3(t-1)} + e^{-(t-1)}\right) - e^{-3t} + e^{-t}$$

Problem 19

Start by finding the Laplace transform of both sides.

$$\mathcal{L}\{y''\} - 9\mathcal{L}\{y\} = \mathcal{L}\{u_6(t)\} + \mathcal{L}\{\delta(t-3)\}$$

$$s^2Y - s\cdot y(0) - y'(0) - 9Y = \frac{e^{-6s}}{s} + e^{-3s}$$

$$s^2Y - 9Y = \frac{e^{-6s}}{s} + e^{-3s}$$

$$Y(s^2 - 9) = \frac{e^{-6s}}{s} + e^{-3s}$$

$$Y = \frac{e^{-6s}}{s(s+3)(s-3)} + \frac{e^{-3s}}{(s+3)(s-3)}$$

$$y = \mathcal{L}^{-1}\left\{e^{-6s}\frac{1}{s(s+3)(s-3)}\right\} + \mathcal{L}^{-1}\left\{e^{-3s}\frac{1}{(s+3)(s-3)}\right\}$$

PFD #1:

$$\frac{1}{s(s+3)(s-3)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s-3}$$

$$A = -\frac{1}{9}, B = \frac{1}{18}, C = \frac{1}{18}$$

PFD #2:

$$\frac{1}{(s+3)(s-3)} = \frac{A}{s+3} + \frac{B}{s-3}$$

$$A = -\frac{1}{6}, B = \frac{1}{6}$$

$$y = \mathcal{L}^{-1}\left\{e^{-6s}\left(-\frac{1}{9}\cdot\frac{1}{s} + \frac{1}{18}\cdot\frac{1}{s+3} + \frac{1}{18}\cdot\frac{1}{s-3}\right)\right\} + \mathcal{L}^{-1}\left\{e^{-3s}\left(-\frac{1}{6}\cdot\frac{1}{s+3} + \frac{1}{6}\cdot\frac{1}{s-3}\right)\right\}$$

$$y = u_6(t)f_1(t-6) + u_3(t)f_2(t-3)$$

$$f_1(t) = \mathcal{L}^{-1}\left\{\left(-\frac{1}{9}\cdot\frac{1}{s} + \frac{1}{18}\cdot\frac{1}{s+3} + \frac{1}{18}\cdot\frac{1}{s-3}\right)\right\} = -\frac{1}{9} + \frac{1}{18}e^{-3t} + \frac{1}{18}e^{3t}$$

$$f_2(t) = \mathcal{L}^{-1}\left\{-\frac{1}{6}\cdot\frac{1}{s+3} + \frac{1}{6}\cdot\frac{1}{s-3}\right\} = -\frac{1}{6}e^{-3t} + \frac{1}{6}e^{3t}$$

$$y = u_6(t)\left(-\frac{1}{9} + \frac{1}{18}e^{-3(t-6)} + \frac{1}{18}e^{3(t-6)}\right) + u_3(t)\left(-\frac{1}{6}e^{-3(t-3)} + \frac{1}{6}e^{3(t-3)}\right)$$

Problem 20

a. True

b. False – $A = \sqrt{(C_1)^2 + (C_2)^2} = \sqrt{2^2 + 2^2} = \sqrt{8}$

c. False – It will remain constant.

Problem 21

a.

$$r^2 + 4r + 9 = 0$$

$$r = -2 \pm \sqrt{5}i$$

$$x(t) = C_1 e^{-2t} \cos(\sqrt{5}t) + C_2 e^{-2t} \sin(\sqrt{5}t)$$

$$x(0) = 0:$$
$$0 = C_1 \cos(0) + C_2 \sin(0)$$
$$C_1 = 0$$

$$x(t) = C_2 e^{-2t} \sin(\sqrt{5}t)$$

$$x'(t) = \sqrt{5}C_2 e^{-2t} \cos(\sqrt{5}t) - 2C_2 e^{-2t} \sin(\sqrt{5}t)$$

$$x'(0) = 1:$$
$$1 = \sqrt{5}C_2 \cos(0) - 2C_2 \sin(0)$$
$$C_2 = 1/\sqrt{5}$$

$$x(t) = \frac{1}{\sqrt{5}} e^{-2t} \sin(\sqrt{5}t)$$

b. $m = 1, \gamma = 4, k = 9$

Underdamped

c.

$$x = 0$$

$$0 = \frac{1}{\sqrt{5}} e^{-2t} \sin(\sqrt{5}t)$$

$$0 = \sin(\sqrt{5}t)$$

$$\sqrt{5}t = n\pi$$

$$t = \frac{n\pi}{\sqrt{5}}$$

Problem 22

$$m = 1, L = \frac{5}{2}, \gamma = 0, \quad x(0) = 1, x'(0) = 0$$

a. Use Hooke's Law:

$$mg = KL$$
$$(1)(10) = k\left(\frac{5}{2}\right)$$
$$k = 4$$

$$x'' + 4x = 0, x(0) = 1, x'(0) = 0$$

b.

$$r^2 + 4 = 0$$

$$r^2 = \pm 2i$$

$$x(t) = C_1 \cos(2t) + C_2 \sin(2t)$$

$$x(0) = 1:$$
$$1 = C_1 \cos(0) + C_2 \sin(0)$$
$$C_1 = 1$$

$$x(t) = \cos(2t) + C_2 \sin(2t)$$

$$x'(t) = -2\sin(2t) + 2C_2 \cos(2t)$$

$$x'(0) = 0:$$
$$0 = -2\sin(0) + 2C_2 \cos(0)$$
$$0 = C_2$$

$$x(t) = \cos(2t)$$

c. The system is undamped, so the lowest point is equal to the amplitude.

$$A = \sqrt{(C_1)^2 + (C_2)^2} = \sqrt{1^2 + 0^2} = 1\ m \text{ below the equilibrium position}$$

d.

$$x(t) = 0$$

$$\cos(2t) = 0$$

$$2t = \frac{(2n+1)\pi}{2}$$

$$t = \frac{(2n+1)\pi}{4}$$