LionTutors®

MATH140

Final Exam: New Material

Sample Test - Solutions

Problem 1

Use L'Hospital's Rule:

$$\lim_{x\to 1} \frac{e^x}{\frac{1}{x}} = \frac{e}{1} = e$$

Problem 2

Evaluate the limit using L'Hospital's Rule.

Simplify first…

$$e^{\lim_{x\to 0} \frac{\ln(\sin x + e^x)}{x}}$$

Apply L.H. Rule…

$$e^{\lim_{x\to 0} \frac{\frac{1}{\sin x + e^x}(\cos x + e^x)}{1}}$$

$$e^{\lim_{x\to 0} \frac{\cos x + e^x}{\sin x + e^x}} = e^2$$

Problem 3

Use L'Hospital's Rule:

$$\lim_{x\to 0} \frac{f'(x)}{5\cos(5x)} = \frac{f'(0)}{5\cos(0)} = \frac{10}{5} = 2$$

Problem 4:

$d = \sqrt{(x-2)^2 + (y-0)^2}$

$D = (x-2)^2 + y^2$

Constraint:

$8x^2 + y^2 = 56$

$y^2 = 56 - 8x^2$

$D = (x-2)^2 + 56 - 8x^2$

$D' = 2(x-2) - 16x$

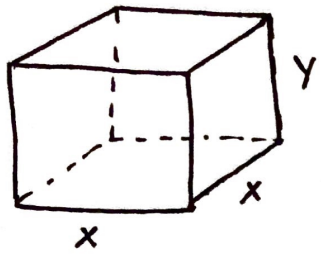
$D' = 2x - 4 - 16x$

$0 = -14x - 4$

$14x = -4$

$x = \frac{-4}{14} \rightarrow \boxed{x = \frac{-2}{7}}$

Problem 5



$V = 8$

$x \cdot x \cdot y = 8$

$y = \frac{8}{x^2}$

$C = 5x^2 + 4(3)xy + 1x^2$

$C = 6x^2 + 12xy$

$C = 6x^2 + 12x\left(\frac{8}{x^2}\right)$

$C = 6x^2 + 96x^{-1}$

$C' = 12x - 96x^{-2}$

$0 = 12x - \frac{96}{x^2}$

$12x^3 = 96$

$x^3 = 8$

$x = 2 \rightarrow y = \frac{8}{x^2} = \frac{8}{4} = 2$

Dimensions: $2\,m \times 2m \times 2m$

Verify: Second Derivative Test

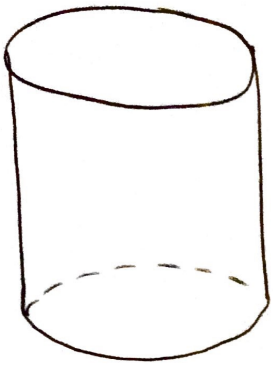
$C' = 12x - 96x^{-2}$

$C'' = 12 + \frac{96}{x^3}$

$C''(2) = 12 + \frac{96}{8} > 0$

Since $C'(2) = 0$, $C''(2) > 0$ then $x = 2$ is a min.

Problem 6



$V = 20\pi$

$\pi r^2 h = 20\pi$

$r^2 h = 20$

$h = \frac{20}{r^2}$

$C = 10\pi r^2 + 10\pi r^2 + 8 \cdot 2\pi r h$

$C = 20\pi r^2 + 16\pi r h$

$C = 20\pi r^2 + 16\pi r \left(\frac{20}{r^2}\right)$

$C = 20\pi r^2 + 320\pi r^{-1}$

$C' = 40\pi r - 320\pi r^{-2}$

$0 = 40\pi r - \frac{320\pi}{r^2}$

$\frac{320\pi}{r^3} = 40\pi$

$40r^3 = 320$

$r^3 = 8$

$r = 2 \rightarrow h = \frac{20}{(2)^2} = 5$

$C = 20\pi (2)^2 + 16\pi (2)(5)$

$C = 80\pi + 160\pi$

$\boxed{C = \$240\pi}$

Verify:

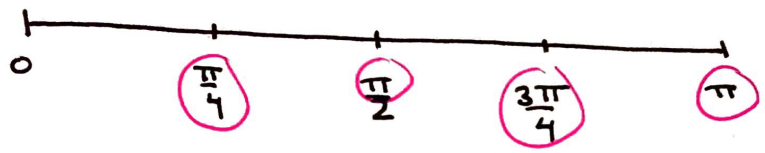
$C' = 40\pi r - 320\pi r^{-2}$

$C'' = 40\pi + \frac{320\pi}{r^3}$

$C''(2) = 40\pi + \frac{320\pi}{8} > 0 \rightarrow$ MIN.

Problem 7:

$$\Delta x = \frac{b-a}{n} = \frac{\pi - 0}{4} = \frac{\pi}{4}$$



$$A \approx \frac{\pi}{4}\left(f\left(\frac{\pi}{4}\right) + f(\pi) + f\left(\frac{3\pi}{4}\right) + f(\pi)\right)$$

$$\approx \frac{\pi}{4}\left(\sin^2\left(\frac{\pi}{4}\right) + \sin^2\left(\frac{\pi}{2}\right) + \sin^2\left(\frac{3\pi}{4}\right) + \sin^2(\pi)\right)$$

$$\approx \frac{\pi}{4}\left(\frac{1}{2} + 1 + \frac{1}{2} + 0\right)$$

$$\approx \frac{\pi}{4}(2)$$

$$\approx \boxed{\frac{\pi}{2}}$$

Problem 8

Sketch the graph of a function that satisfies the given criteria. Clearly label all extrema and infelction points.

$f(0) = 4, f(3) = 1, f(5) = 3$

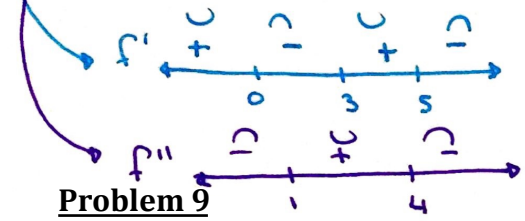
$f'(0) = f'(3) = f'(5) = 0$

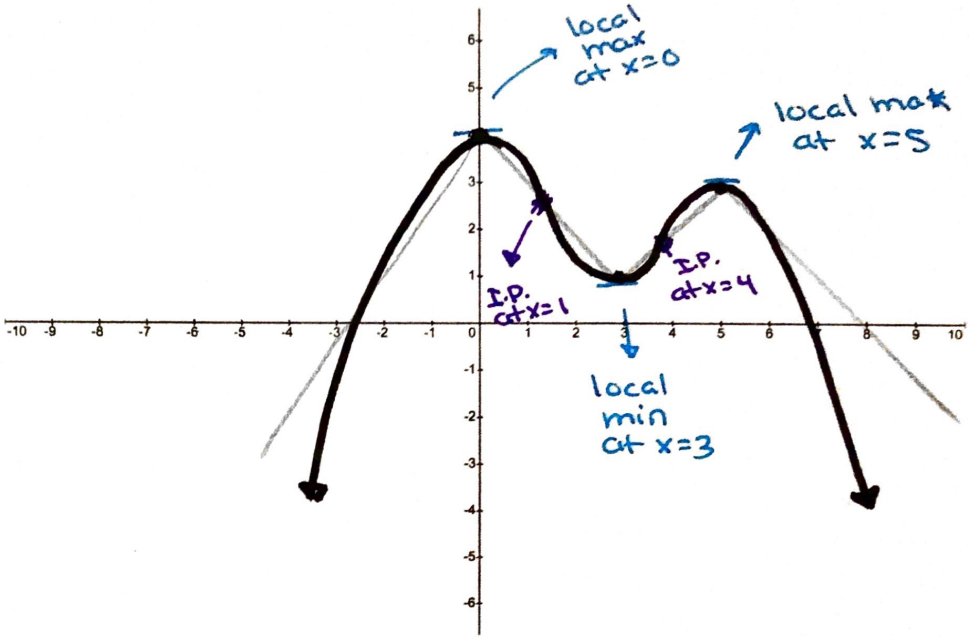
$f'(x) > 0$ for $x < 0$ and $3 < x < 5$

$f'(x) < 0$ for $0 < x < 3$ and $x > 5$

$f''(x) > 0$ for $1 < x < 4$

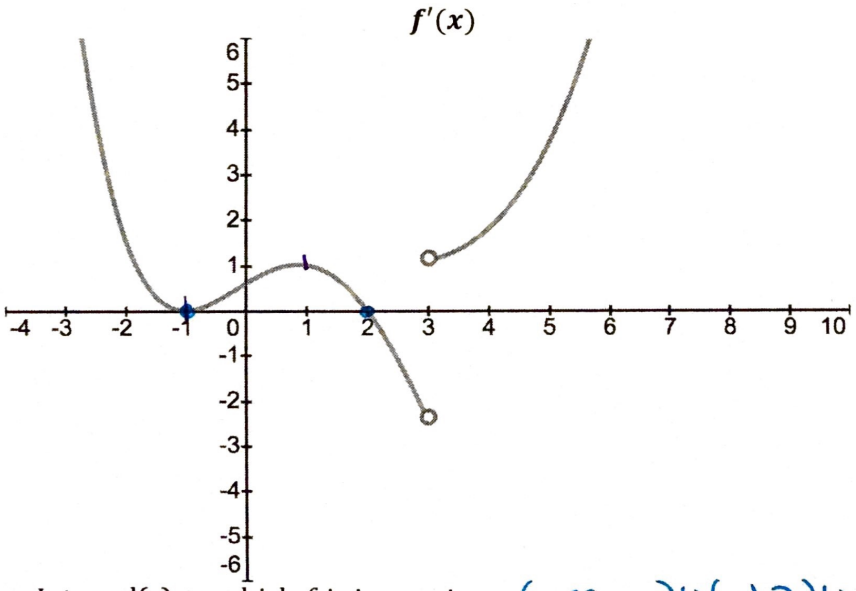
$f''(x) < 0$ for $x < 1$ and $x > 4$



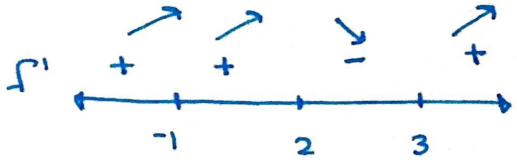


Problem 9

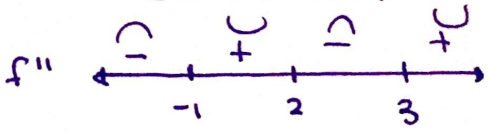
Shown below is the graph of the first derivative, $f'(x)$, of a function $f(x)$ that is continuous on $(-\infty, \infty)$. Use the graph to answer the questions below:



(above / below)



(inc. / dec)



Interval(s) on which f is increasing: $(-\infty, -1) \cup (-1, 2) \cup (3, \infty)$

x-value(s) where f has a local maximum: $x = 2$

x-value(s) where f has a local minimum: $x = 3$

Interval(s) where f is concave up: $(-1, 2) \cup (3, \infty)$

x-value(s) where f has an inflection point: $x = -1, 2, 3$

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Problem 10:

detailed work on the next 2 pages...

NOTE: If no answer exists for any of these questions, write "NONE".

a) State the domain of the function: $x \neq 0$ OR $(-\infty, 0) \cup (0, \infty)$

b) State all x and y intercepts: $(-2, 0), (1, 0)$

c) State all equations of horizontal asymptotes: $y = 1$

d) State all equations of vertical asymptotes: $x = 0$

e) List the interval(s) where $f(x)$ is decreasing: $(-\infty, 0) \cup (4, \infty)$

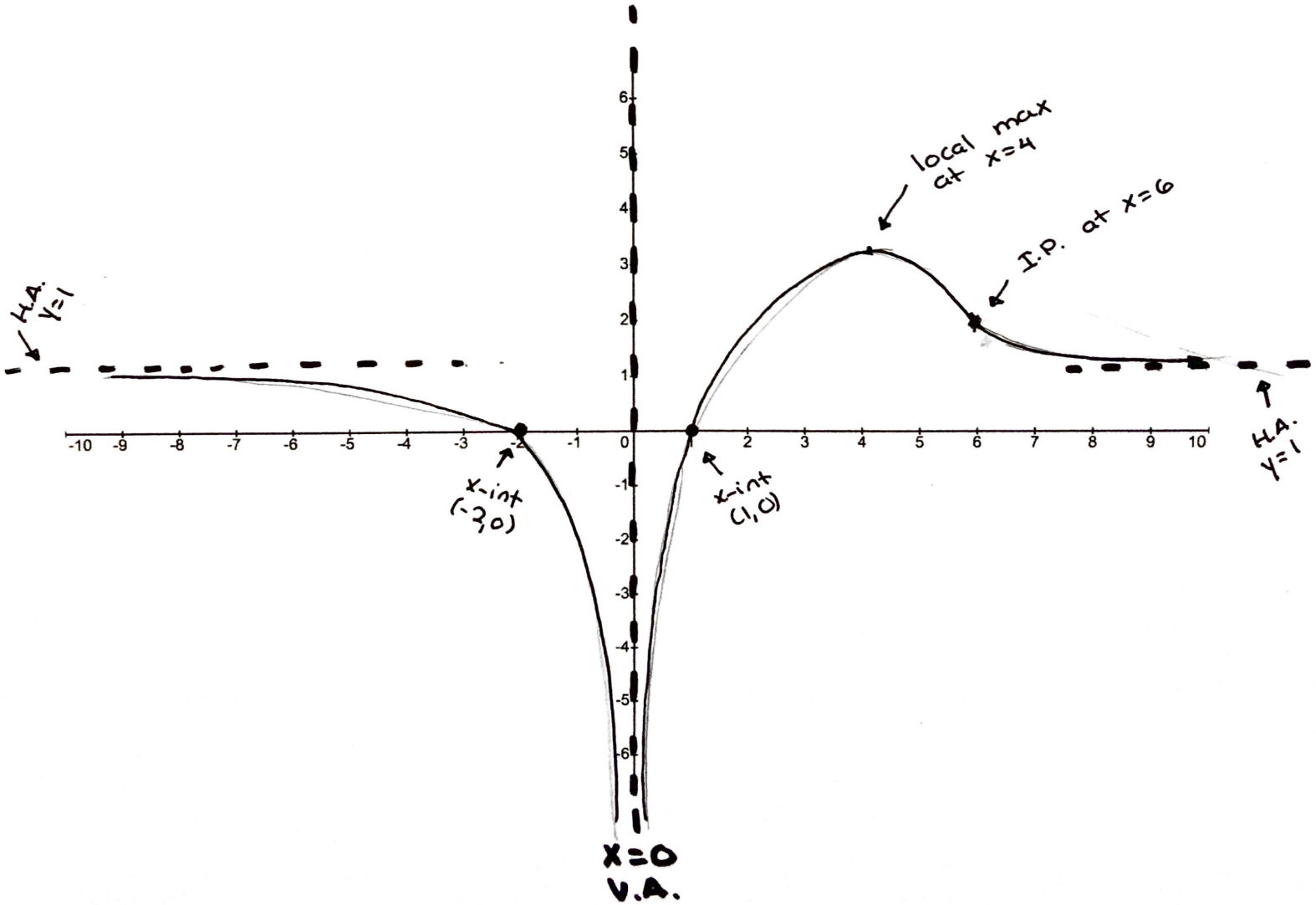
f) State the x-values of all local maxima: $x = 4$

g) State the x-values of all local minima: NONE

h) List the interval(s) where $f(x)$ is concave up: $(6, \infty)$

i) State the x-value(s) of all inflection points: $x = 6$

j) Sketch the graph of f. Label all intercepts, asymptotes, local extrema, and inflections points.



$$f(x) = \frac{x^2 + x - 2}{x^2}$$

domain: $x^2 \neq 0 \rightarrow x \neq 0$

x-int: $0 = \frac{x^2 + x - 2}{x^2} \rightarrow 0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x = -2, \; x = 1$

$(-2, 0), (1, 0)$

y-int: $y = \frac{0 + 0 - 2}{0}$

$\rightarrow x \neq 0$, so no y-int.

H.A. $y = 1$ → look at powers

V.A. $x = 0$ → set den. = 0

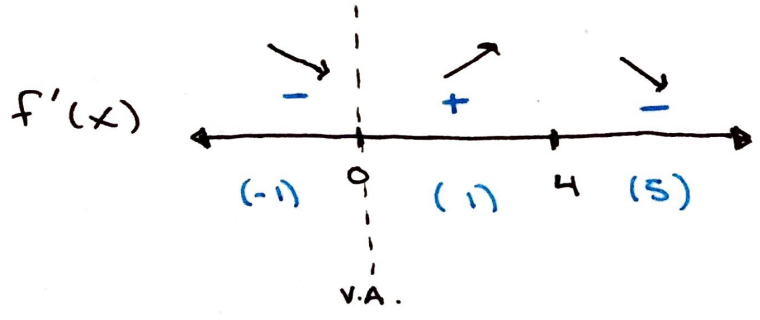
$$f'(x) = \frac{4 - x}{x^3}$$

$4 - x = 0$

$x = 4$

$x^3 = 0$

$x = 0$

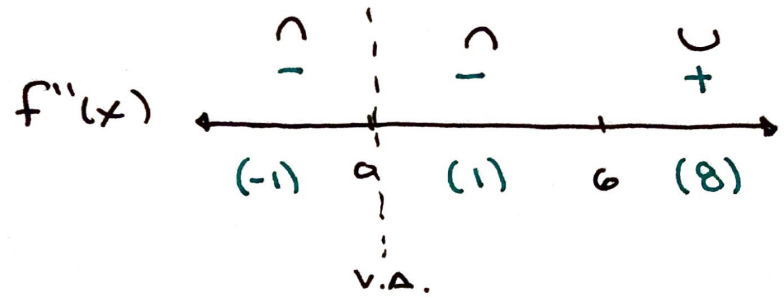


dec: $(-\infty, 0) \cup (4, \infty)$

local max: $x = 4$

local min: none ($x = 0$ is not in domain)

$$f''(x) = \frac{2x-12}{x^4} \rightarrow 2x-12=0 \quad x=6 \qquad x^4=0 \quad x=0$$



concave up: $(6, \infty)$

I.P.: $x = 6$

Problem 11

Use rules of integration:

$$\int (2^x + 4x^{-1/2} + x^{-1} + \cos x + 5)\,dx$$

$$= \frac{2^x}{\ln 2} + \frac{4x^{1/2}}{\frac{1}{2}} + \ln|x| + \sin x + 5x + C$$

$$= \frac{2^x}{\ln 2} + 8\sqrt{x} + \ln|x| + \sin x + 5x + C$$

Problem 12

$$f'(x) = 1 + 2x - \sin x$$

$$f(x) = \int (1 + 2x - \sin x)\,dx$$

$$f(x) = x + x^2 + \cos x + C$$

Plug in $(0,5)$ to solve for C:

$$5 = 0 + (0)^2 + \cos(0) + C$$

$$5 = 1 + C$$

$$C = 4$$

$$f(x) = x + x^2 + \cos x + 4$$

Problem 13

$$v(t) = \int a(t)\,dt$$

$$v(t) = \int \cos t \; dt$$

$$v(t) = \sin t + c$$

Use $v(0) = 2$ to solve for c:

$$2 = \sin(0) + c$$
$$2 = 0 + c$$
$$c = 2$$

$$v(t) = \sin t + 2$$

$$s(t) = \int v(t)\,dt$$

$$s(t) = \int (\sin t + 2)\,dt$$

$$s(t) = -\cos t + 2t + D$$

Use $s(0) = 3$ to solve for D:

$$3 = -\cos(0) + 0 + D$$

$$3 = -1 + D$$

$$D = 4$$

$$s(t) = -\cos t + 2t + 4$$

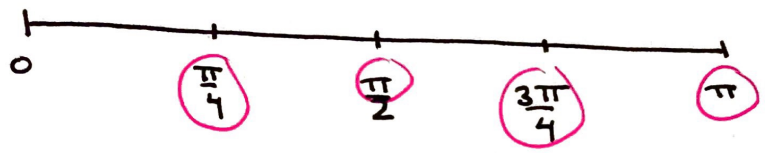
$$s(\pi) = -\cos(\pi) + 2\pi + 4$$

$$= -(-1) + 2\pi + 4$$

$$= \boxed{5 + 2\pi}$$

Problem 14

$$\Delta x = \frac{b-a}{n} = \frac{\pi - 0}{4} = \frac{\pi}{4}$$



$$A \approx \frac{\pi}{4}\left(f\left(\frac{\pi}{4}\right) + f(\pi) + f\left(\frac{3\pi}{4}\right) + f(\pi)\right)$$

$$\approx \frac{\pi}{4}\left(\sin^2\left(\frac{\pi}{4}\right) + \sin^2\left(\frac{\pi}{2}\right) + \sin^2\left(\frac{3\pi}{4}\right) + \sin^2(\pi)\right)$$

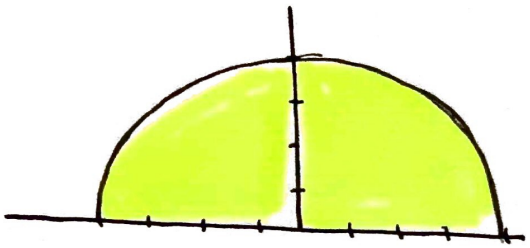
$$\approx \frac{\pi}{4}\left(\frac{1}{2} + 1 + \frac{1}{2} + 0\right)$$

$$\approx \frac{\pi}{4}(2)$$

$$\approx \boxed{\frac{\pi}{2}}$$

Problem 15

$y = \sqrt{16 - x^2}$ is the top half of a circle with radius = 4



$$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(4)^2 = \boxed{8\pi}$$

Problem 16

Use FTC

$$\tan^{-1}(2)\cdot(0) - \tan^{-1}(x^3)\cdot 3x^2$$

$$= \boxed{-3x^2\tan^{-1}(x^3)}$$

Problem 17

Foil, then divide by x to simplify:

$$\int \left(\frac{x^2}{x} + \frac{2x}{x} + \frac{1}{x}\right) dx$$

$$\int (x + 2 + x^{-1})\, dx$$

$$\frac{x^2}{2} + 2x + \ln x \Big]_2^8$$

$$\left(\frac{64}{2} + 16 + \ln 8\right) - \left(\frac{4}{2} + 4 + \ln 2\right)$$

$$48 + \ln 8 - 6 - \ln 2$$

$$42 + \ln 8 - \ln 2$$

$\ln 8 - \ln 2 = \ln\left(\frac{8}{2}\right) = \ln 4$

$$42 + \ln 4$$

Problem 18

$$\int_0^5 (250 - 10t)\,dt$$

$$= 250t - 5t^2 \Big]_0^5$$

$$= \left(250(5) - 5(25)\right) - 0$$

$$= \boxed{1125 \text{ liters/min}}$$

Problem 19

$$\Delta x = \frac{7-2}{n} = \frac{5}{n}, \quad x_i = \frac{5}{n}i + 2, \quad f(x) = \ln(x)$$

$$f(x_i) = \ln\left(\frac{5}{n}i + 2\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(f(x_i)\cdot \Delta x\right)$$

$$\boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \left(\ln\left(\frac{5}{n}i + 2\right)\right)\left(\frac{5}{n}\right)}$$

Problem 20: A

Problem 21: C

Problem 22: B

Problem 23

a. $\int \frac{e^x}{(1-e^x)^2}\,dx$

$u = 1 - e^x$

$du = -e^x\,dx$

$dx = \frac{du}{-e^x}$

$= \int \frac{\cancel{e^x}}{u^2} \frac{du}{\cancel{-e^x}}$

$= \int -u^{-2}\,du$

$= -\frac{u^{-1}}{-1} = \frac{1}{u} = \boxed{\frac{1}{1-e^x} + C}$

b. $\int_1^2 x\sqrt{x-1}\,dx$

$u = x-1$

$du = dx$

$\int_1^2 x \cdot u^{1/2}\,du$ ⟶ $u = x-1$

$x = u+1$

$\int (u+1)u^{1/2}\,du$

$\int (u^{3/2} + u^{1/2})\,du$

$\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}$

$\frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2}\Big]_1^2$

$\left[\frac{2}{5}(1)^{5/2} + \frac{2}{3}(1)^{3/2}\right] - [0 + 0]$

$\frac{2}{5} + \frac{2}{3} = \frac{6}{15} + \frac{10}{15} = \boxed{\frac{16}{15}}$

c. $\int \tan(3x)\,dx$

$= \int \frac{\sin(3x)}{\cos(3x)}\,dx$

$u = \cos(3x)$

$du = -3\sin(3x)\,dx$

$dx = \frac{du}{-3\sin(3x)}$

$= \int \frac{\sin(3x)}{u} \cdot \frac{du}{-3\sin(3x)}$

$= \int -\frac{1}{3}u^{-1}\,du$

$= -\frac{1}{3}\ln|u| + C$

$= \boxed{-\frac{1}{3}\ln|\cos(3x)| + C}$

d.

$$\int \frac{2x+2}{x^2+4}\,dx$$

$$\int \frac{2x}{x^2+4}\,dx + \int \frac{2}{x^2+4}\,dx$$

$u = x^2+4$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$$\int \frac{2x}{u}\,\frac{du}{2x}$$

$$\int u^{-1}$$

$$\ln|u|$$

$$\ln(x^2+4)$$

$$\int 2 \cdot \frac{1}{x^2+4}\,dx$$

$$2\left(\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right)$$

$$\tan^{-1}\left(\frac{x}{2}\right)$$

$$\boxed{\ln(x^2+4) + \tan^{-1}\left(\frac{x}{2}\right) + C}$$

Problem 24

a. $\int_0^2 (5 + f'(4x))dx$

$= \int_0^2 5\,dx + \int_0^2 f'(4x)\,dx$

$= 5x\Big]_0^2 + \frac{1}{4}f(4x)\Big]_0^2$

$= [5(2) - 5(0)] + \left[\frac{1}{4}f(8) - \frac{1}{4}f(0)\right]$

$= 10 + \frac{1}{4}(0) - \frac{1}{4}(1)$

$= 10 - \frac{1}{4} = \boxed{\frac{39}{4}}$

$u = 4x$

$du = 4dx$

$\int f'(u)\frac{du}{4}$

$= \frac{1}{4}\int f'(u)du$

$= \frac{1}{4}f(u)$

$= \frac{1}{4}f(4x)$

b. $\int_0^5 f'(2x)e^{f(2x)}\,dx$

$u = f(2x)$

$du = f'(2x)\cdot 2\,dx$

$dx = \frac{du}{2f'(2x)}$

$\int f'(2x)e^{u}\frac{du}{2f'(2x)}$

$\int \frac{1}{2}e^{u}\,du = \frac{1}{2}e^{u} = \frac{1}{2}e^{f(2x)}\Big]_0^5$

$= \frac{1}{2}e^{f(10)} - \frac{1}{2}e^{f(0)} = \frac{1}{2}e^{5} - \frac{1}{2}e^{1} = \boxed{\frac{e^5 - e}{2}}$

c. $\int_0^8 \frac{f'(x)}{1+(f(x))^2}\,dx$

$u = f(x)$

$du = f'(x)dx$

$dx = \frac{du}{f'(x)}$

$\int \frac{f'(x)}{1+u^2}\frac{du}{f'(x)}$

$\int \frac{1}{1+u^2}\,du$

$\tan^{-1}(u)$

$\tan^{-1}(f(x))\Big]_0^8$

$\tan^{-1}(f(8)) - \tan^{-1}(f(0))$

$\tan^{-1}(0) - \tan^{-1}(1) = 0 - \frac{\pi}{4} = \boxed{-\frac{\pi}{4}}$

d. $\int_0^2 \frac{f'(5x)}{f(5x)}\,dx$

$u = f(5x)$

$du = f'(5x) \cdot 5\,dx$

$dx = \frac{du}{5f'(5x)}$

$\int \frac{f'(5x)}{u} \cdot \frac{du}{5f'(5x)}$

$\int \frac{1}{5} u^{-1}\,du$

$\frac{1}{5} \ln|u|$

$\frac{1}{5} \ln|f(5x)| \Big]_0^2$

$\frac{1}{5} \ln|f(10)| - \frac{1}{5} \ln|f(0)|$

$\frac{1}{5} \ln(5) - \frac{1}{5} \ln(1)^0$

$\boxed{\frac{1}{5} \ln(5)}$