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MATH 022 – Final (Review of Exam 1, 2, & 3) – Practice Exam Solutions

Problem #1

a. x-intercepts: (-1, 0), (2.5, 0)
   y-intercept (approximated): $\left(0, \frac{1}{3}\right)$

b. Positive: [-4, -1), (-1, 2.5)
   Negative: (2.5, 4]

c. Average rate of change:

   i. $\frac{0-4}{-1-(-4)} = -\frac{4}{3}$ decreasing

   ii. $\frac{1-0}{1-(-1)} = \frac{1}{2}$ increasing

   iii. $\frac{1-1}{2-1} = 0$ constant

   iv. $\frac{-3-1}{4-2} = -2$ decreasing

Problem #2

$f(x) = x^2 + 2x - 4$

$f(x+h) = (x+h)^2 + 2(x+h) - 4$
$f(x+h) = x^2 + 2xh + h^2 + 2x + 2h - 4$

$$Difference\ quotient = \frac{x^2 + 2xh + h^2 + 2x + 2h - 4 - (x^2 + 2x - 4)}{h}$$

$$= \frac{x^2 + 2xh + h^2 + 2x + 2h - 4 - x^2 - 2x + 4}{h}$$

$$= \frac{2xh + h^2 + 2h}{h}$$

$$= 2x + h + 2$$

Problem #3

a. $p(1) = 1^2 + 5(1) = \ 6$

1 day after a rumor begins, approximately 6% of the town has heard some version of the rumor.

b. $p(t) = t^2 + 5t$

$p(t + h) = (t + h)^2 + 5(t + h)$
$p(t + h) = t^2 + 2th + h^2 + 5t + 5h$

$$Difference\ quotient = \frac{t^2 + 2th + h^2 + 5t + 5h - (t^2 + 5t)}{h}$$

$$= \frac{t^2 + 2th + h^2 + 5t + 5h - t^2 - 5t}{h}$$

$$= \frac{2th + h^2 + 5h}{h}$$

$$= 2t + h + 5$$

c. $2(2) + (0.1) + 5 = 9.1$

During the time period from 2 days to 2.1 days after the rumor begins, the percentage of the town that has heard the rumor increases by an average of approximately 9.1% per day.

Problem #4

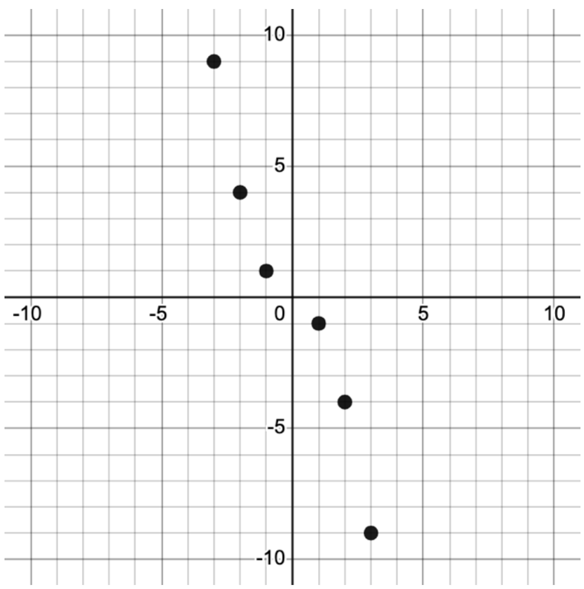
a. $f(x) = \frac{3x}{5-x^3}$

$$f(-x) = \frac{3(-x)}{5-(-x)^3} = -\left(\frac{3x}{5+x^3}\right)$$

$$-\left(\frac{3x}{5+x^3}\right) \neq \left(\frac{3x}{5-x^3}\right) \neq -\left(\frac{3x}{5-x^3}\right)$$

Neither: $f(-x) \neq f(x) \neq -f(x)$

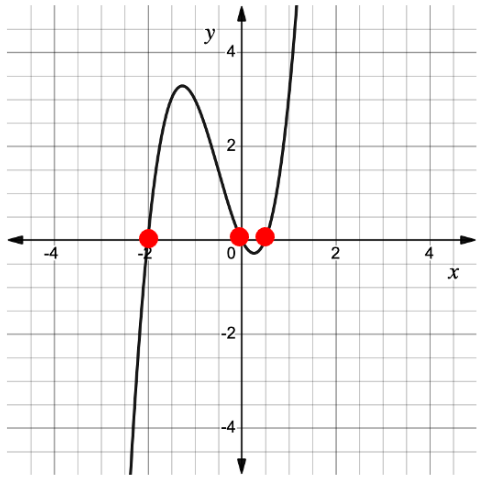
b. **Odd:** $f(x) = y \; and \; f(-x) = -y$



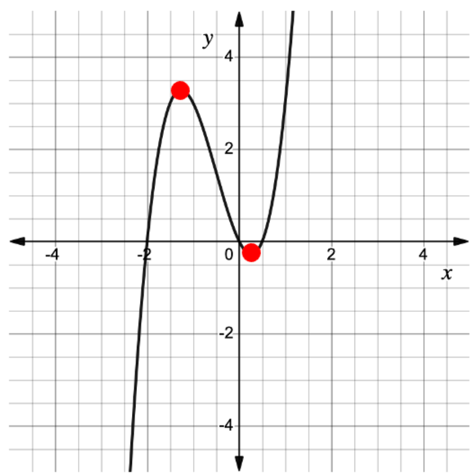
c. **Even:** Symmetric along y-axis

Problem #5

a. x-intercepts: –2, 0, 1



b. 2



c. 3 – The graph can have at most $n - 1$ turning points.

d. $a > 0$
$f(x) \to -\infty \; as \; x \to -\infty$
$f(x) \to \infty \; as \; x \to \infty$

e. $(-\infty, -1.25) \cup (0.25, \infty)$

f. $(-1.25, 0.25)$

g. Absolute maximum: none
Absolute minimum: none
Local maximum: -1.25
Local minimum: 0.25

Problem #6

To find m, plug in any value for t that fits the first piece of the function with its corresponding p value as $f(x)$.

$f(1) = m(8)$
$\boldsymbol{m = 8}$

Then, plug in any value for t that fits the second piece of the function with its corresponding p value as $f(x)$. Isolate one of the missing variables.

$f(x) = a(t-2)^2 + b$
$21 = a(4-2)^2 + b$
$21 = a(2)^2 + b$
$21 = 4a + b$
$b = 21 - 4a$

Next, plug in another value for t that fits the second piece of the function with its corresponding p value as $f(x)$. Substitute b with $21 - 4a$

$f(x) = a(t-2)^2 + b$
$16 = a(5-2)^2 + (21 - 4a)$
$16 = a(3)^2 + 21 - 4a$
$16 = 9a + 21 - 4a$
$-5 = 5a$
$\boldsymbol{a = -1}$

Finally, plug the value of a in to find b.

$b = 21 - 4a$
$b = 21 - 4(-1)$
$\boldsymbol{b = 25}$

Problem #7

Separately divide each term in the numerator by the monomial in the denominator and simplify.

$$\frac{-6x^4}{-3x^2} + \frac{5x^3}{-3x^2} + \frac{9x^2}{-3x^2} - \frac{7x}{-3x^2} + \frac{8}{-3x^2}$$

$$2x^2 - \frac{5}{3}x - 3 + \frac{7}{3x} - \frac{8}{3x^2}$$

Problem #8

Use the "Division by Polynomials" steps listed on Page 3 of the review packet (long division form).

$$
\begin{array}{r l}
 & 6x^2 + 4x - 15 \\
x^2 + 0x + 2 \,\big) & 6x^4 + 4x^3 - 3x^2 + 7x - 5 \\
 & -(6x^4 + 0x^3 + 12x^2) \downarrow \\
 & \quad 4x^3 - 15x^2 + 7x \\
 & \quad -(4x^3 + 0x^2 + 8x) \\
 & \qquad -15x^2 - x - 5 \\
 & \qquad -(-15x^2 - 0x - 30) \\
 & \qquad\qquad - x + 25
\end{array}
$$

$$6x^2 + 4x - 15 + \frac{-x + 25}{x^2 + 2}$$

Problem #9

Use the "Synthetic Division" steps listed on Page 4 of the review packet.

–3	2	0	5	–1	4
	↓	–6	18	–69	210
	2	–6	23	–70	214

$$2x^3 - 6x^2 + 23x - 70 + \frac{214}{x + 3}$$

Problem #10

Use the Remainder Theorem.

$$f(-3) = 4(-3)^2 + 5(-3) - 6$$

$$f(-3) = 36 - 15 - 6$$

$$f(-3) = 15$$

The remainder is 15.

Problem #11

a. Look at the x-intercepts on the graph to find the zeros of $p(x)$.

Zero	Even/Odd Multiplicity	Multiplicity
−4	Even $p(x)$ **does not cross** over x-axis at $x = -4$	2 $p(x)$ **levels off less** near the x-axis at $x = -4$ vs. $x = 1$; must be a **lower** even multiplicity
1	Even $p(x)$ **does not cross** over x-axis at $x = 1$	4 $p(x)$ **levels off more** near the x-axis at $x = 1$ vs. $x = -4$; must be a **higher** even multiplicity
5	Odd $p(x)$ **does cross** over x-axis at $x = 5$	1 The sum of the multiplicities equals the degree , $\boldsymbol{n = 7}$.

b. Use the leading coefficient, the zeros, and the multiplicities to write the complete factored form.

$$p(x) = 0.001(x+4)^2(x-1)^4(x-5)$$

Problem #12

a. Since $f(x)$ has a complex zero at $c = i$, we know that the conjugate, $c = -i$, must also be a zero.

We can start by factoring out $(x - c)$ for each of the known zeros:

$$(x - i)(x + i) = x^2 - i^2 = x^2 + 1$$

Use long-division form to divide $f(x)$ by $(x^2 + 1)$.

$$\begin{array}{r|l} & x^2 + 4x - 2 \\ \hline x^2 + 0x + 1 & x^4 + 4x^3 - x^2 + 4x - 2 \\ & -(x^4 + 0x^3 + x^2) \\ \hline & \quad 4x^3 - 2x^2 + 4x \\ & \quad -(4x^3 + 0x^2 + 4x) \\ \hline & \qquad -2x^2 + 0x - 2 \\ & \qquad -(-2x^2 - 0x - 2) \\ \hline & \qquad\qquad 0 \end{array}$$

Set the quotient equal to zero to find the other zeros of the function.

$$x^2 + 4x - 2 = 0$$

Use the quadratic formula to solve.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-2)}}{2(1)} = \frac{-4 \pm \sqrt{24}}{2} = \frac{-4 \pm 2\sqrt{6}}{2} = -\frac{4}{2} \pm \frac{2\sqrt{6}}{2} = -2 \pm \sqrt{6}$$

Complex nonreal zeros: $i, -i$
Real zeros: $-2 - \sqrt{6}, -2 + \sqrt{6}$

b. Use the zeros to write the complete factored form. Because the leading coefficient is 1, it can be omitted.

$$f(x) = (x - i)(x + i)\left[x - (-2 - \sqrt{6})\right]\left[x - \left(-2 + \sqrt{6}\right)\right]$$

$$f(x) = (x - i)(x + i)(x + 2 + \sqrt{6})(x + 2 - \sqrt{6})$$

Problem #13

a. $D: [0, \infty)$ – Time can't be negative.

b. Solve for $C(t)$ when $t = 0$ to find the vertical intercept.

$$C(0) = \frac{4{,}000(0)}{0^2 + 400} = 0$$

Solve for t when $C(t) = 0$ to find the horizontal intercept.

$$0 = \frac{4{,}000t}{t^2 + 400}$$

$$0 = 4{,}000t$$

$$t = 0$$

The vertical and horizontal intercept occurs at (0,0).

c. The degree of the numerator is less than the degree of the denominator, so there is a horizontal asymptote at $y = 0$.

There are no real values of t for which $q(t) = 0$, so there are no vertical asymptotes.

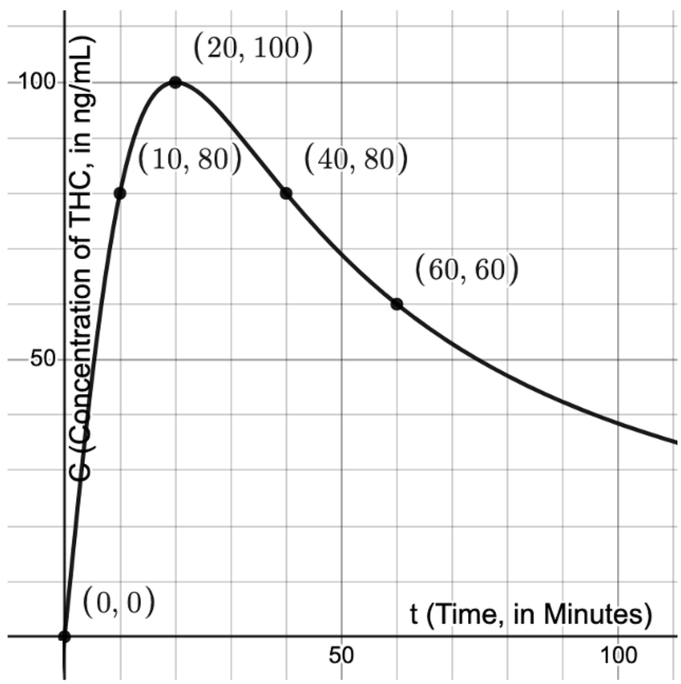
d. Solve for $C(t)$ when $t = 10$.

$$C(10) = \frac{4{,}000(10)}{10^2 + 400} = 80 \text{ ng/mL}$$

e. Make a table of values.

t	$C(t)$
20	$\frac{4{,}000(20)}{20^2+400} = 100$
40	$\frac{4{,}000(40)}{40^2+400} = 80$
60	$\frac{4{,}000(60)}{60^2+400} = 60$

Use the intercept and the known points to sketch a graph.



f. Initially, Mike's blood THC concentration increases rapidly. Then, after about 20 minutes, it begins to decrease, leveling off as it approaches zero.

Problem #14
First, rearrange the inequality so that one side is zero.

$$\frac{9}{2x-4} - \frac{6}{x-3} \leq 0$$

Next, multiply both fractions to get a common denominator. Note that we are just multiplying each fraction by 1, so we haven't changed the inequality.

$$\left(\frac{9}{2x-4}\right)\left(\frac{x-3}{x-3}\right) - \left(\frac{6}{x-3}\right)\left(\frac{2x-4}{2x-4}\right) \leq 0$$

$$\frac{9x-27}{(2x-4)(x-3)} - \frac{12x-24}{(2x-4)(x-3)} \leq 0$$

Now that we have a common denominator, we can subtract the fractions.

$$\frac{9x-27-(12x-24)}{(2x-4)(x-3)} \leq 0$$

$$\frac{9x-27-12x+24}{(2x-4)(x-3)} \leq 0$$

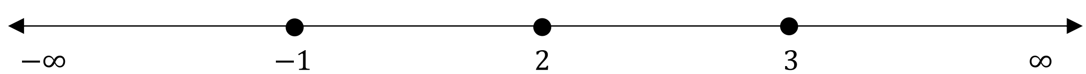
$$\frac{-3x-3}{(2x-4)(x-3)} \leq 0$$

Set the numerator and denominator each equal to zero to find the boundary numbers.

$$-3x-3=0$$
$$x=-1$$

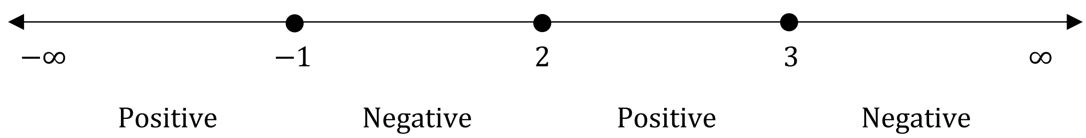
$$(2x-4)(x-3)=0$$
$$x=2, x=3$$

Use the boundary numbers to split the number line into intervals.



Test a value within each interval to determine whether the rational function is positive or negative on each interval.

Test Value	$\boldsymbol{f(x)}$	**Positive or Negative**
–2	$\frac{-3(-2)-3}{(2(-2)-4)(-2-3)} = \frac{3}{40}$	Positive
0	$\frac{-3(0)-3}{(2(0)-4)(0-3)} = \frac{-3}{12} = -\frac{1}{4}$	Negative
2.5	$\frac{-3(2.5)-3}{(2(2.5)-4)(2.5-3)} = \frac{-10.5}{-0.5} = 21$	Positive
4	$\frac{-3(4)-3}{(2(4)-4)(4-3)} = -\frac{15}{4}$	Negative



Find the intervals where the inequality is less than or equal to zero (the negative intervals, including the boundary numbers that set the numerator equal to zero).

$$\frac{-3x-3}{(2x-4)(x-3)} \leq 0$$

Remember, values that set the denominator equal to zero are never included in the solution set.

$$[-1, 2) \cup (3, \infty)$$

Problem #15
First, rearrange the inequality so that one side is zero.

$$\frac{x^2 - 3x - 10}{2 - x} - 3 < 0$$

Next, multiply to get a common denominator. Note that we are just multiplying by 1, so we haven't changed the inequality.

$$\frac{x^2 - 3x - 10}{2 - x} - 3\left(\frac{2 - x}{2 - x}\right) < 0$$

$$\frac{x^2 - 3x - 10}{2 - x} - \frac{6 - 3x}{2 - x} < 0$$

Now that we have a common denominator, we can subtract the fractions.

$$\frac{x^2 - 3x - 10 - (6 - 3x)}{2 - x} < 0$$

$$\frac{x^2 - 3x - 10 - 6 + 3x}{2 - x} < 0$$

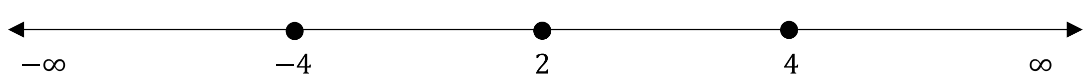
$$\frac{x^2 - 16}{2 - x} < 0$$

Set the numerator and denominator each equal to zero to find the boundary numbers.

$$x^2 - 16 = 0$$
$$(x - 4)(x + 4) = 0$$
$$x = 4, x = -4$$

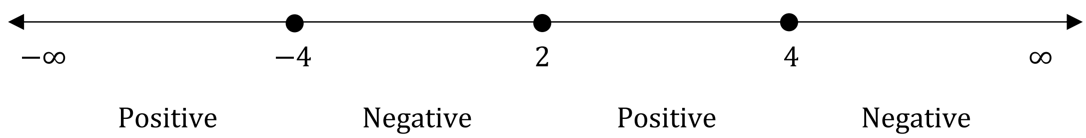
$$2 - x = 0$$
$$x = 2$$

Use the boundary numbers to split the number line into intervals.



Test a value within each interval to determine whether the rational function is positive or negative on each interval.

Test Value	$\boldsymbol{f(x)}$	**Positive or Negative**
–5	$\frac{(-5)^2 - 16}{2-(-5)} = \frac{9}{7}$	Positive
1	$\frac{1^2 - 16}{2-1} = \frac{-15}{1} = -15$	Negative
3	$\frac{3^2 - 16}{2-3} = \frac{-7}{-1} = 7$	Positive
5	$\frac{5^2 - 16}{2-5} = \frac{9}{-3} = -3$	Negative



Find the intervals where the inequality is less than zero (the negative intervals, not including the boundary numbers).

$$\frac{x^2 - 16}{2 - x} < 0$$

$$(-4, 2) \cup (4, \infty)$$

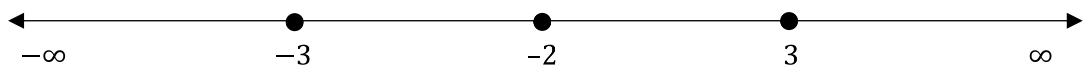
Problem #16
First, rearrange the inequality so that one side is zero.

$$x^3 + 2x^2 - 9x - 18 < 0$$

Next, set the function equal to zero to find the boundary numbers. Factor by grouping to solve for the zeros.

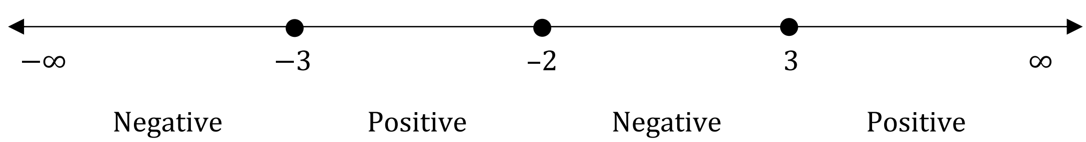
$$x^3 + 2x^2 - 9x - 18 = 0$$
$$x^2(x + 2) - 9(x + 2) = 0$$
$$(x^2 - 9)(x + 2) = 0$$
$$(x - 3)(x + 3)(x + 2) = 0$$
$$x = 3.\ x = -3,\ x = -2$$

Use the boundary numbers to split the number line into intervals.



Test a value within each interval to determine whether the rational function is positive or negative on each interval.

Test Value	$\boldsymbol{f(x)}$	**Positive or Negative**
–4	$(-4 - 3)(-4 + 3)(-4 + 2) = -14$	Negative
–2.5	$(-2.5 - 3)(-2.5 + 3)(-2.5 + 2) = 1.375$	Positive
1	$(1 - 3)(1 + 3)(1 + 2) = -24$	Negative
4	$(4 - 3)(4 + 3)(4 + 2) = 42$	Positive



Find the intervals where the inequality is less than zero (the negative intervals, not including the boundary numbers).

$$x^3 + 2x^2 - 9x - 18 < 0$$

$$(-\infty, -3) \cup (-2, 3)$$

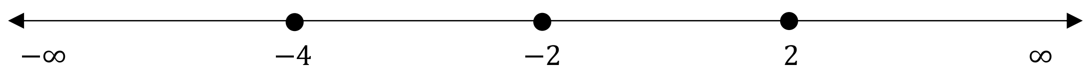
Problem #17
First, rearrange the inequality so that one side is zero.

$$2x^3 + 8x^2 - 8x - 32 \leq 0$$

Next, set the function equal to zero to find the boundary numbers. Factor by grouping to solve for the zeros.

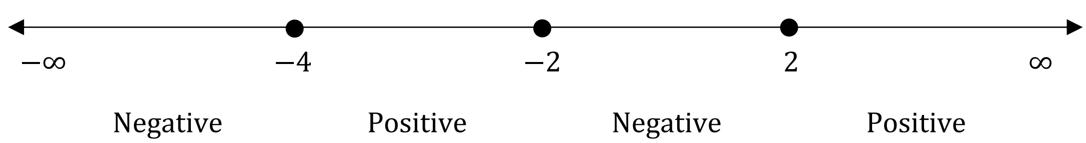
$$2x^2(x + 4) - 8(x + 4) \leq 0$$
$$(2x^2 - 8)(x + 4) \leq 0$$
$$(2x + 4)(x - 2)(x + 4) \leq 0$$
$$x = -2, x = 2, x = -4$$

Use the boundary numbers to split the number line into intervals.



Test a value within each interval to determine whether the rational function is positive or negative on each interval.

Test Value	$\boldsymbol{f(x)}$	**Positive or Negative**
–5	$(2(-5) + 4)(-5 - 2)(-5 + 4) = -42$	Negative
–3	$(2(-3) + 4)(-3 - 2)(-3 + 4) = 10$	Positive
0	$(2(0) + 4)(0 - 2)(0 + 4) = -32$	Negative
3	$(2(3) + 4)(3 - 2)(3 + 4) = 70$	Positive



Find the intervals where the inequality is less than or equal to zero (the negative intervals, including the boundary numbers).

$$2x^3 + 8x^2 - 8x - 32 \leq 0$$

$$(-\infty, -4] \cup [-2, 2]$$

Problem #18

a. Split into three terms and evaluate each using the properties of exponents:

$$-4^{\frac{3}{2}} = -\sqrt{4}^{3} = -2^3 = -8$$

$$-4(2)^{-3} = -4\left(\frac{1}{2^3}\right) = -4\left(\frac{1}{8}\right) = -\frac{4}{8} = -\frac{1}{2}$$

$$(16^4)^{\frac{1}{8}} = 16^{(4)\left(\frac{1}{8}\right)} = 16^{\frac{1}{2}} = \sqrt{16} = 4$$

Combine the simplified terms to evaluate the full expression:

$$-8 - \frac{1}{2} + 4 = -4.5$$

b. Split into two terms and evaluate each using the properties of exponents:

$$10^8 10^{-5} = 10^{8+(-5)} = 10^3 = 1{,}000$$

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

Combine the simplified terms to evaluate the full expression:

$$1{,}000\left(\frac{1}{25}\right) = 40$$

c. Split into two terms and evaluate each using the properties of exponents:

$$\frac{\left(7^{\frac{1}{2}}\right)^{\frac{3}{2}}}{7^{-\frac{1}{4}}} = \frac{7^{\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}}{7^{-\frac{1}{4}}} = \frac{7^{\frac{3}{4}}}{7^{-\frac{1}{4}}} = 7^{\left(\frac{3}{4}-\left(-\frac{1}{4}\right)\right)} = 7^{\frac{4}{4}} = 7$$

$$-3^{10}\left(\frac{3^{-2}}{3^5}\right) = -3^{10}3^{(-2-5)} = -3^{10}3^{-7} = -3^{(10+(-7))} = -3^3 = -27$$

Combine the simplified terms to evaluate the full expression:

$$7 - 27 = -20$$

Problem #19

a. Isolate the radical:

$$\sqrt{8x-7} = 1 + x$$

Square both sides:

$$8x - 7 = (1 + x)^2$$

Solve for x by factoring:

$$8x - 7 = 1 + 2x + x^2$$

$$0 = x^2 - 6x + 8$$

$$0 = (x - 4)(x - 2)$$

Any x-value that sets either factor equal to zero is a possible solution.

$$(x - 4) = 0 \qquad (x - 2) = 0$$

$$x = 4 \qquad x = 2$$

Check for extraneous solutions:

$$\sqrt{8(4) - 7} - 2(4) = 1 - 4 \qquad \sqrt{8(2) - 7} - 2(2) = 1 - 2$$

$$\sqrt{32 - 7} - 8 = -3 \qquad \sqrt{16 - 7} - 4 = -1$$

$$\sqrt{25} = 5 \qquad \sqrt{9} = 3$$

$$5 = 5 \qquad 3 = 3$$

Both solutions satisfy the original equation, so there are two solutions:

$$x = 4,\ x = 2$$

b. Isolate the more complicated radical:

$$\sqrt{2x+3}+3=\sqrt{14x+18}$$

Square both sides:

$$\left(\sqrt{2x+3}+3\right)^2=14x+18$$

Simplify:

$$2x+3+6\sqrt{2x+3}+9=14x+18$$

$$2x+12+6\sqrt{2x+3}=14x+18$$

$$6\sqrt{2x+3}=12x+6$$

$$\sqrt{2x+3}=\frac{12x+6}{6}$$

$$\sqrt{2x+3}=2x+1$$

Square both sides:

$$2x+3=(2x+1)^2$$

Simplify:

$$2x+3=4x^2+4x+1$$

$$0=4x^2+2x-2$$

Solve for x by factoring:

$$0=2(2x^2+x-1)$$

$$0=2(2x-1)(x+1)$$

Any x-value that sets either factor equal to zero is a possible solution.

$$(2x-1)=0 \qquad (x+1)=0$$

$$x=\tfrac{1}{2} \qquad x=-1$$

Check for extraneous solutions:

$$\sqrt{2\left(\frac{1}{2}\right)+3} = \sqrt{14\left(\frac{1}{2}\right)+18} - 3 \qquad \sqrt{2(-1)+3} = \sqrt{14(-1)+18} - 3$$

$$\sqrt{1+3} = \sqrt{7+18} - 3 \qquad \sqrt{-2+3} = \sqrt{-14+18} - 3$$

$$\sqrt{4} = \sqrt{25} - 3 \qquad \sqrt{1} = \sqrt{4} - 3$$

$$2 = 5 - 3 \qquad 1 = 2 - 3$$

$$2 = 2 \qquad 1 \neq -1$$

Since $x = -1$ is an extraneous solution, there is only one real solution:

$$x = \frac{1}{2}$$

Problem #20

a. Use the substitution method:

$$u = x^{\frac{1}{3}}$$

$$\left(x^{\frac{1}{3}}\right)^2 - 4x^{\frac{1}{3}} + 3 = 0$$

$$u^2 - 4u + 3 = 0$$

Solve for u by factoring:

$$(u-3)(u-1) = 0$$

Any u-value that sets either factor equal to zero is a possible solution:

$$(u-3) = 0 \qquad (u-1) = 0$$

$$u = 3 \qquad u = 1$$

Convert u back to x:

$$u = x^{\frac{1}{3}}$$

$$x = u^3$$

$$x = 3^3 \qquad x = 1^3$$

$$x = 27 \qquad x = 1$$

b. Use the substitution method:

$$u = x^{-1} \qquad u^2 = (x^{-1})^2 = x^{-2}$$

$$2u^2 + 3u = 2$$

$$2u^2 + 3u - 2 = 0$$

Solve for u by factoring:

$$(2u - 1)(u + 2) = 0$$

Any u-value that sets either factor equal to zero is a possible solution:

$$(2u - 1) = 0 \qquad (u + 2) = 0$$

$$u = \frac{1}{2} \qquad u = -2$$

Convert u back to x:

$$u = x^{-1}$$

$$u = \frac{1}{x}$$

$$x = \frac{1}{u}$$

$$x = \frac{1}{1/2} \qquad x = \frac{1}{-2}$$

$$x = 2 \qquad x = -\frac{1}{2}$$

Problem #21

a.

$$f(-1) = -2(-1)^{-3} = -2\left(\frac{1}{-1^3}\right) = -2\left(\frac{1}{-1}\right) = 2$$

$$f(0) = -2(0)^{-3} = -2\left(\frac{1}{0^3}\right) = \text{undefined}$$

b. Because n is negative, $f(0)$ is undefined, and the domain includes all real numbers other than 0.

$$D: (-\infty, 0) \cup (0, \infty)$$

Because n is negative, there are no solutions to $f(x) = 0$, so the range includes all real numbers other than 0.

$$R: (-\infty, 0) \cup (0, \infty)$$

c. Use the graph below.

Increasing: : $(-\infty, 0) \cup (0, \infty)$

Decreasing: None

d. Use the graph below.

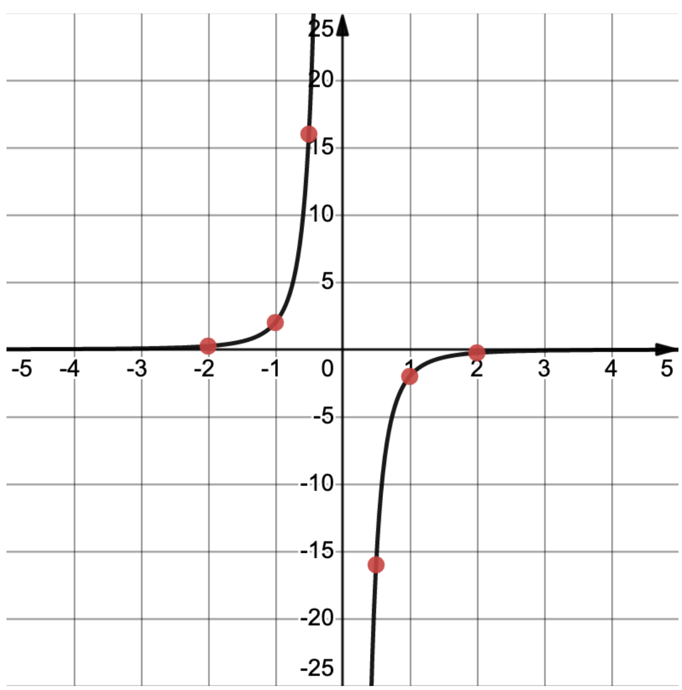
As $x \to \infty$, $f(x) \to 0$

As $x \to -\infty$, $f(x) \to 0$

e. Graph features:

- Because n is a negative integer, $x \neq 0$ and $f(x) \neq 0$.
    - Vertical asymptote at $x = 0$
    - Horizontal asymptote at $y = 0$
- Because n is odd:
    - It is possible for $f(x)$ to have both negative and positive solutions.
    - The graph is on both sides of the x-axis.
- Because $a > 0$, the graph is reflected across the x-axis.
    - If $x > 0 \rightarrow y < 0$
    - If $x < 0 \rightarrow y > 0$

x	$f(x)$
-2	$-2(-2)^{-3} = -2\left(\frac{1}{-2^3}\right) = -2\left(\frac{1}{-8}\right) = \frac{1}{4}$
-1	$-2(-1)^{-3} = -2\left(\frac{1}{-1^3}\right) = -2\left(\frac{1}{-1}\right) = 2$
$-\frac{1}{2}$	$-2\left(-\frac{1}{2}\right)^{-3} = -2(-2^3) = -2(-8) = 16$
$\frac{1}{2}$	$-2\left(\frac{1}{2}\right)^{-3} = -2(2^3) = -2(8) = -16$
1	$-2(1)^{-3} = -2\left(\frac{1}{1^3}\right) = -2\left(\frac{1}{1}\right) = -2$
2	$-2(2)^{-3} = -2\left(\frac{1}{2^3}\right) = -2\left(\frac{1}{8}\right) = -\frac{1}{4}$



Problem #22

a. Replace $f(x)$ with y:

$$y = \frac{4x+1}{3-x}$$

Solve for x:

$$y(3-x) = 4x+1$$

$$3y - xy = 4x+1$$

$$3y - 1 = 4x + xy$$

$$3y - 1 = x(4+y)$$

$$\frac{3y-1}{4+y} = x$$

Switch x and y:

$$y = \frac{3x-1}{4+x}$$

Replace y with $f^{-1}(x)$:

$$\boldsymbol{f^{-1}(x) = \frac{3x-1}{4+x}}$$

Check your answer:

$$(f \circ f^{-1})(x) = x$$

$$x = \frac{4\left(\frac{3x-1}{4+x}\right)+1}{3-\left(\frac{3x-1}{4+x}\right)}$$

$$x\left(3-\left(\frac{3x-1}{4+x}\right)\right) = 4\left(\frac{3x-1}{4+x}\right)+1$$

$$3x - \left(\frac{x(3x-1)}{4+x}\right) = \frac{4(3x-1)}{4+x}+1$$

$$3x - \left(\frac{3x^2-x}{4+x}\right) = \frac{12x-4}{4+x}+1$$

$$3x(4+x) - (3x^2-x) = 12x - 4 + 1(4+x)$$

$$12x + 3x^2 - 3x^2 + x = 12x - 4 + 4 + x$$

$$13x = 13x$$

$$x = x$$

b.

$$(f \circ g)(x) = f\big(g(x)\big) = f(2x - 6)$$

$$= \frac{4(2x-6)+1}{3-(2x-6)}$$

$$= \frac{8x - 24 + 1}{3 - 2x + 6}$$

$$= \mathbf{\frac{8x - 23}{9 - 2x}}$$

c.

$$(g \circ f)(x) = g\big(f(x)\big) = g\left(\frac{4x+1}{3-x}\right)$$

$$= 2\left(\frac{4x+1}{3-x}\right) - 6$$

$$= \frac{2(4x+1)}{3-x} - 6$$

$$= \frac{8x+2}{3-x} - 6\left(\frac{3-x}{3-x}\right)$$

$$= \frac{8x+2}{3-x} - \frac{6(3-x)}{3-x}$$

$$= \frac{8x+2}{3-x} - \frac{18-6x}{3-x}$$

$$= \frac{8x + 2 - (18 - 6x)}{3-x}$$

$$= \frac{8x + 2 - 18 + 6x}{3-x}$$

$$= \frac{14x - 16}{3-x}$$

Problem #23

a.

$$f(2) = -1$$

$$g(2) = 2$$

$$4(-1) - 7(2) = -4 - 14 = \mathbf{-18}$$

b.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$\frac{f(-8)}{g(-8)} = \frac{9}{-3} = -\mathbf{3}$$

c.

$$(f \circ g)(x) = f\big(g(x)\big)$$

$$g(5) = -6$$

$$f(-6) = \mathbf{7}$$

d.

$$(f \circ f)(x) = f\big(f(x)\big)$$

$$f(0) = 1$$

$$f(1) = \mathbf{0}$$

e.

$$(fg)(x) = f(x) * g(x)$$

$$f(4) * g(4) = (-3)(-3) = \mathbf{9}$$

Problem #24

a. Any x-value can be plugged in:

$$D: (-\infty, \infty)$$

Regardless of the x-value, $f(x) > 0$:

$$R: (0, \infty)$$

b. Each time x increases by 1, the previous value of $f(x)$ is multiplied by a:

$$a = \frac{2}{5}$$

$$0 < a < 1$$

f is always decreasing

c. f is always decreasing but never reaches 0:

Horizontal asymptote at $y = 0$

No vertical asymptotes

d. When $x = 0$, $f(x) = C = 4$:

y-intercept: $(0, 4)$

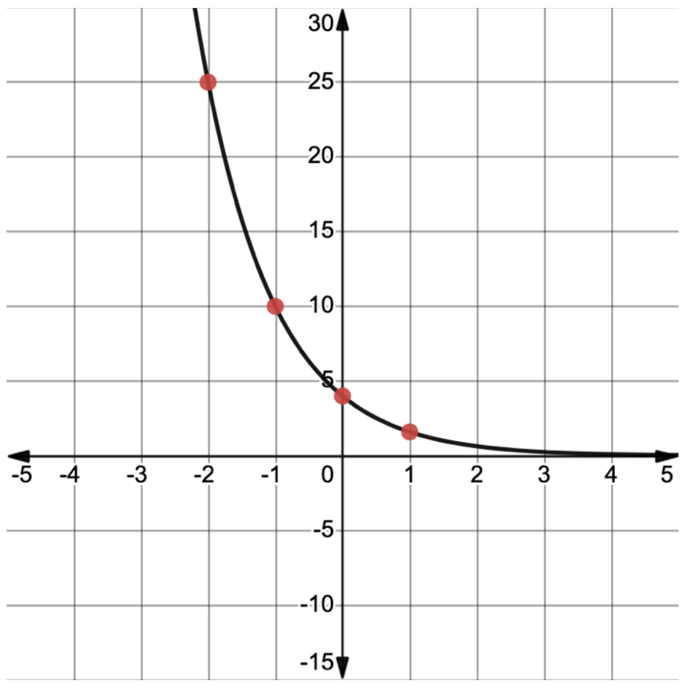
The graph of f never touches the x-axis, so there are no x-intercepts.

e. Yes, f is a one-to-one function because no two x-values produce the same y-value.

Yes, f has an inverse because it is a one-to-one function.

f.

x	$f(x)$
-2	$4\left(\frac{2}{5}\right)^{-2} = 4\left(\frac{5^2}{2}\right) = 4\left(\frac{25}{4}\right) = 25$
-1	$4\left(\frac{2}{5}\right)^{-1} = 4\left(\frac{5^1}{2}\right) = 4\left(\frac{5}{2}\right) = 10$
1	$4\left(\frac{2}{5}\right)^{1} = 4\left(\frac{2}{5}\right) = \frac{8}{5}$



Problem #25

a. Convert hours to minutes:

$$4 \text{ hours} * 60 \text{ minutes/hour} = 240 \text{ minutes}$$

Plug $t = 240$ into the function:

$$S(240) = 40 + \frac{1}{5}(240 - 35) = 40 + \frac{1}{5}(205) = 40 + 41 = \mathbf{81}$$

b. Yes – $S(t)$ is a linear function, which means it is a one-to-one function and has an inverse.

c. For the function S, the domain is the number of minutes a student studied and the range is the expected exam score.

For the inverse function S^{-1}, the domain is the expected exam score and the range is the number of minutes a student studied.

d. Solve the function for t:

$$S = 40 + \frac{1}{5}(t - 35)$$

$$S = 40 + \frac{1}{5}t - 7$$

$$S = 33 + \frac{1}{5}t$$

$$S - 33 = \frac{1}{5}t$$

$$5(S - 33) = t$$

$$5S - 165 = t$$

$$\boldsymbol{t = 5S - 165}$$

Plug 90 into the inverse function:

$$t = 5(90) - 165 = 450 - 165 = \mathbf{285}$$

To have an expected exam score of 90, a student must study for 285 minutes.

Problem #26

$$P = 10{,}000$$

$$r = .04$$

a. If interest is compounded annually, $n = 1$.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A(t) = 10{,}000\left(1 + \frac{.04}{1}\right)^{1t}$$

$$A(t) = 10{,}000(1.04)^{t}$$

b. If interest is compounded quarterly, $n = 4$.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A(t) = 10{,}000\left(1 + \frac{.04}{4}\right)^{4t}$$

$$A(t) = 10{,}000(1.01)^{4t}$$

c. If interest is compounded continuously, use the equation:

$$A = Pe^{rt}$$

$$A(t) = 10{,}000e^{.04t}$$

Problem #27

a. The half-life is $k = 8$ hours and the initial amount is $C = 10$ mg.

$$A(t) = C\left(\frac{1}{2}\right)^{\frac{t}{k}}$$

$$A(t) = 10\left(\frac{1}{2}\right)^{\frac{t}{8}}$$

b. Plug $t = 4$ into the function:

$$A(4) = 10\left(\frac{1}{2}\right)^{\frac{4}{8}} = 10\left(\frac{1}{2}\right)^{\frac{1}{2}} = 10\sqrt{\frac{1}{2}} \approx \mathbf{7.1\ mg}$$

A person has about 7.1 mg of Adderall in their system 4 hours after taking a 10 mg dosage.

c. It takes 8 hours for the amount to reach half of its previous value:

$$10\left(\frac{1}{2}\right) = 5 \text{ mg remaining after 8 hours}$$

$$5\left(\frac{1}{2}\right) = 2.5 \text{ mg remaining after 16 hours}$$

$$2.5\left(\frac{1}{2}\right) = 1.25 \text{ mg remaining after } \textbf{24 hours}$$

Problem #28

a.

$$-2\ln 4x = -6$$

$$\ln 4x = 3$$

$$e^{\ln 4x} = e^3$$

$$4x = e^3$$

$$x = \frac{1}{4}e^3$$

b.

$$\log 2x = 4$$

$$10^{\log 2x} = 10^4$$

$$2x = 10{,}000$$

$$x = 5{,}000$$

c.

$$3\log_2(9x) = -6$$

$$\log_2(9x) = -2$$

$$2^{\log_2(9x)} = 2^{-2}$$

$$9x = \frac{1}{2^2}$$

$$9x = \frac{1}{4}$$

$$x = \frac{1}{36}$$

d.

$$3(2^x) = 24$$

$$2^x = 8$$

$$x = 3$$

e.

$$e^{2x} = e^{\frac{1}{3}}$$

$$\ln e^{2x} = \ln e^{\frac{1}{3}}$$

$$2x = \frac{1}{3}$$

$$x = \frac{1}{6}$$

Problem #29

a. You can only find the logarithm of positive values.

$$3x - 12 > 0$$

$$3x > 12$$

$$x > 4$$

$$(4, \infty)$$

b. You can only find the logarithm of positive values.

$$x^2 - 25 > 0$$

$$x^2 > 25$$

$$x > 5 \qquad\qquad x < -5$$

$$(-\infty, -5) \cup (5, \infty)$$

c. You can only find the logarithm of positive values, **and** you can only take the square root of values greater than or equal to zero.

$$5 - \sqrt{x+3} > 0 \qquad \text{and} \qquad x + 3 \geq 0$$

$$-\sqrt{x+3} > -5 \qquad\qquad x \geq -3$$

$$\sqrt{x+3} < 5$$

$$x + 3 < 25$$

$$x < 22$$

$$[-3, 22)$$

d. You can only find the logarithm of positive values, but exponential functions are always positive.

$$1^x > 0$$

$$(-\infty, \infty)$$