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MATH 022 – Final Exam (New Material) – Practice Exam Solutions

Problem #1

a. You can only find the logarithm of positive values.

$$3x - 12 > 0$$

$$3x > 12$$

$$x > 4$$

$$(4, \infty)$$

b. You can only find the logarithm of positive values.

$$x^2 - 25 > 0$$

$$x^2 > 25$$

$$x > 5 \qquad\qquad x < -5$$

$$(-\infty, -5) \cup (5, \infty)$$

c. You can only find the logarithm of positive values, **and** you can only take the square root of values greater than or equal to zero.

$$5 - \sqrt{x+3} > 0 \qquad \text{and} \qquad x + 3 \geq 0$$

$$-\sqrt{x+3} > -5 \qquad\qquad x \geq -3$$

$$\sqrt{x+3} < 5$$

$$x + 3 < 25$$

$$x < 22$$

$$[-3, 22)$$

d. You can only find the logarithm of positive values, but exponential functions are always positive.

$$1^x > 0$$

$$(-\infty, \infty)$$

Problem #2

In terms of the common logarithm:

$$2\left(\frac{\log 12}{\log 8}\right) - \sqrt{\frac{\log 5}{\log 3}}$$

Approximated to 3 decimal places:

$$2\left(\frac{1.079}{0.903}\right) - \sqrt{\frac{0.699}{0.477}} \approx 1.179$$

Problem #3

In terms of the natural logarithm:

$$\frac{\ln 5}{\ln 2} \div \frac{\ln 14}{\ln 2} = \frac{\ln 5}{\ln 2} \times \frac{\ln 2}{\ln 14} = \frac{\ln 5}{\ln 14}$$

Approximated to 3 decimal places:

$$\frac{1.609}{2.639} \approx 0.610$$

Problem #4

a.

$$-2\ln 4x = -6$$

$$\ln 4x = 3$$

$$e^{\ln 4x} = e^3$$

$$4x = e^3$$

$$x = \frac{1}{4}e^3$$

b.

$$\log 2x = 4$$

$$10^{\log 2x} = 10^4$$

$$2x = 10{,}000$$

$$x = 5{,}000$$

c.

$$3\log_2(9x) = -6$$

$$\log_2(9x) = -2$$

$$2^{\log_2(9x)} = 2^{-2}$$

$$9x = \frac{1}{2^2}$$

$$9x = \frac{1}{4}$$

$$x = \frac{1}{36}$$

d.

$$3(2^x) = 24$$

$$2^x = 8$$

$$x = 3$$

e.

$$e^{2x} = e^{\frac{1}{3}}$$

$$\ln e^{2x} = \ln e^{\frac{1}{3}}$$

$$2x = \frac{1}{3}$$

$$x = \frac{1}{6}$$

Problem #5

a.

$$\log_3 9 + \log_3 x^3 + \log_3 y^5$$

$$2 + 3\log_3 x + 5\log_3 y$$

b.

$$\log x^2 - \log\left(10y^5\sqrt{z^3}\right)$$

$$\log x^2 - \left(\log 10 + \log y^5 + \log z^{\frac{3}{2}}\right)$$

$$\log x^2 - \log 10 - \log y^5 - \log z^{\frac{3}{2}}$$

$$2\log x - 1 - 5\log y - \frac{3}{2}\log z$$

c.

$$\ln(x-2)^2 - \ln(x^2+2)$$

$$2\ln(x-2) - \ln(x^2+2)$$

d.

$$\log_4\left(\frac{6-x}{y^2z^{10}}\right)^{\frac{1}{5}}$$

$$\frac{1}{5}\log_4\left(\frac{6-x}{y^2z^{10}}\right)$$

$$\frac{1}{5}\log_4(6-x) - \frac{1}{5}\log_4(y^2z^{10})$$

$$\frac{1}{5}\log_4(6-x) - \frac{1}{5}\log_4 y^2 + \frac{1}{5}\log_4 z^{10}$$

$$\frac{1}{5}\log_4(6-x) - \frac{2}{5}\log_4 y + 2\log_4 z$$

Problem #6

a.

$$\log_4\left(\frac{x^5x^4}{\sqrt[3]{x}}\right)$$

$$\log_4\left(\frac{x^5x^4}{x^{\frac{1}{3}}}\right)$$

$$\log_4\left(x^{5+4-\frac{1}{3}}\right)$$

$$\log_4 x^{\frac{26}{3}}$$

$$\log_4 \sqrt[3]{x^{26}}$$

b.

$$\log x^3 + \log y^{\frac{1}{2}} - \log z^{\frac{1}{4}}$$

$$\log\left(\frac{x^3 y^{\frac{1}{2}}}{z^{\frac{1}{4}}}\right)$$

$$\log\left(\frac{x^3\sqrt{y}}{\sqrt[4]{z}}\right)$$

c.

$$\frac{1}{5}\log_2[(x+3)(x-3)]$$

$$\frac{1}{5}\log_2(x^2-9)$$

$$\log_2(x^2-9)^{\frac{1}{5}}$$

$$\log_2\sqrt[5]{x^2-9}$$

d.

$$\ln 2^4 + \ln\sqrt{x^3} - \ln x$$

$$\ln 16 + \ln x^{\frac{3}{2}} - \ln x$$

$$\ln\frac{16x^{\frac{3}{2}}}{x}$$

$$\ln 16x^{\frac{3}{2}-1}$$

$$\ln 16x^{\frac{1}{2}}$$

$$\ln 16\sqrt{x}$$

Problem #7

a.

$$\log[(2x+1)(x-3)] = \log(4x+2)$$

$$\log(2x^2-5x-3) = \log(4x+2)$$

$$10^{\log(2x^2-5x-3)} = 10^{\log(4x+2)}$$

$$2x^2-5x-3 = 4x+2$$

$$2x^2-9x-5 = 0$$

$$(2x+1)(x-5) = 0$$

$$x = -\frac{1}{2} \qquad x = 5$$

You can only find the logarithm of positive values, so $\boldsymbol{x = 5}$ is the only solution.

b.

$$\ln x^3 x^2 = 5$$

$$\ln x^5 = 5$$

$$e^{\ln x^5} = e^5$$

$$x^5 = e^5$$

$$x = e$$

c.

$$\log_2 2x4x = 5$$

$$\log_2 8x^2 = 5$$

$$2^{\log_2 8x^2} = 2^5$$

$$8x^2 = 32$$

$$x^2 = 4$$

$$x = 2 \qquad x = -2$$

You can only find the logarithm of positive values, so $\boldsymbol{x = 2}$ is the only solution.

d.

$$\log_5(x-1)^2 = \log_5(2x+3)$$

$$5^{\log_5(x-1)^2} = 5^{\log_5(2x+3)}$$

$$(x-1)^2 = 2x+3$$

$$x^2 - 2x + 1 = 2x + 3$$

$$x^2 - 4x - 2 = 0$$

$$a = 1, \qquad b = -4, \qquad c = -2$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6}$$

$$x = 2 + \sqrt{6} \qquad\qquad x = 2 - \sqrt{6}$$

You can only find the logarithm of positive values, so $\boldsymbol{x = 2 + \sqrt{6}}$ is the only solution.

e.

$$\log_2(x-1) - \log_2(x-8) = 3$$

$$\log_2 \frac{x-1}{x-8} = 3$$

$$2^{\log_2 \frac{x-1}{x-8}} = 2^3$$

$$\frac{x-1}{x-8} = 8$$

$$x - 1 = 8(x-8)$$

$$x - 1 = 8x - 64$$

$$-7x = -63$$

$$x = 9$$

Problem #8

a.

$$4\left(\frac{1}{2}\right)^x = 6$$

$$\left(\frac{1}{2}\right)^x = \frac{3}{2}$$

$$\log\left(\frac{1}{2}\right)^x = \log\frac{3}{2}$$

$$x\log\frac{1}{2} = \log\frac{3}{2}$$

$$x = \frac{\log\frac{3}{2}}{\log\frac{1}{2}}$$

b.

$$\ln e^{6x} = \ln 3^{x-1}$$

$$6x = (x-1)\ln 3$$

$$6x = x\ln 3 - \ln 3$$

$$6x - x\ln 3 = -\ln 3$$

$$x(6-\ln 3) = -\ln 3$$

$$x = -\frac{\ln 3}{6-\ln 3}$$

c.

$$5x + 7 = 2x^2$$

$$0 = 2x^2 - 5x - 7$$

$$0 = (2x-7)(x+1)$$

$$x = \frac{7}{2} \qquad x = -1$$

Problem #9

Write in quadratic form:

$$(e^x)^2 + 4(e^x) - 5 = 0$$

Use the substitution method:

$$u = e^x$$

$$u^2 + 4u - 5 = 0$$

Solve for u:

$$(u + 5)(u - 1) = 0$$

$$u = -5 \qquad u = 1$$

Solve for x:

$$u = e^x$$

$$e^x = -5 \qquad e^x = 1$$

$$\ln e^x = \ln 1$$

$$x = 0$$

Exponential functions are always positive, so $e^x = -5$ has no solutions. The only solution is $\boldsymbol{x = 0}$.

Problem #10

Write in quadratic form:

$$(\log_2 x)^2 + (\log_2 x) - 12 = 0$$

Use the substitution method:

$$u = \log_2 x$$

$$u^2 + u - 12 = 0$$

Solve for u:

$$(u + 4)(u - 3) = 0$$

$$u = -4 \qquad u = 3$$

Solve for x:

$$u = \log_2 x$$

$$\log_2 x = -4 \qquad \log_2 x = 3$$

$$2^{\log_2 x} = 2^{-4} \qquad 2^{\log_2 x} = 2^3$$

$$x = \frac{1}{16} \qquad x = 8$$

Problem #11

a.

$$2^{4x} \leq 16$$

$$\log_2 2^{4x} \leq \log_2 16$$

$$4x \leq 4$$

$$x \leq 1$$

$$(-\infty, 1]$$

b.

$$2 \log x > 1$$

$$\log x > \frac{1}{2}$$

$$10^{\log x} > 10^{\frac{1}{2}}$$

$$x > \sqrt{10}$$

$$(\sqrt{10}, \infty)$$

Problem #12

a. **Logistic** – Initially, the function increases exponentially (it slopes upward more rapidly as the y-values get higher). After a certain point, the data levels out. This type of data is modeled with a logistic function.

b. **Exponential decay** – The data *decreases* exponentially (it slopes downward rapidly when the y-values are high, and it slopes downward more gradually as the y-values get lower), which means it can be represented by an exponential decay model.

c. **Exponential growth** – The data *increases* exponentially (it slopes upward more rapidly as the y-values get higher), which means it can be represented by an exponential growth model.

d. **Logarithmic** – The data increases at a decreasing rate (it levels out as the x-values get higher). This type of data is modeled with a logarithmic function.

Problem #13

a. **Linear** – Each time x increases by 1, y decreases by 6. This is a constant rate of change, modeled by a linear function.

b. **Exponential decay** – Each time x increases by 5, y decreases by less than it did the previous time. The data *decreases* exponentially (it slopes downward rapidly when the y-values are high, and it slopes downward more gradually as the y-values get lower), which means it can be represented by an exponential decay model.

c. **Logarithmic** – Each time x increases by 500, y increases by less than it did the previous time. The data increases at a decreasing rate (it levels out as the x-values get higher). This type of data is modeled with a logarithmic function.

d. **Logistic** – At first, each time x increases by 5, y increases by more than it did the previous time. After a certain point, as x continues to increase by 5, y increases by less than it did the previous time.

   Initially, the function increases exponentially (it slopes upward more rapidly as the y-values get higher). After a certain point, the data levels out. This type of data is modeled with a logistic function.

e. **Exponential growth** – Each time x increases by 0.1, y increases by more than it did the previous time. The data *increases* exponentially (it slopes upward more rapidly as the y-values get higher), which means it can be represented by an exponential growth model.

Problem #14

a. **Logarithmic** – The first hour of training per week would significantly increase the runner's speed. However, as the runner gets in better shape, each additional hour would make less of a difference on the speed.

   The data increases at a decreasing rate (it levels out as the x-values get higher). This type of data is modeled with a logarithmic function.

b. **Exponential** – If the birthrate remains constant, each person is having the same number of children, on average. Each year, the population is multiplied by the same value.

   As the population grows, the number being multiplied becomes larger, causing the population to grow more quickly. This can be modeled by an exponential function.

c. **Logistic** – At first, the virus would spread more rapidly as more people become infected (for example, say each infected person comes into contact with 10 people, and each of those 10 people come into contact with 10 more people…) After a certain point, as more people gain immunity to the virus, it would start to spread more slowly.

   Initially, the function is increasing exponentially. After a certain point, the data levels out. This type of data is modeled with a logistic function.