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MATH 022 – Exam 3 – Practice Exam Solutions

Problem #1

a. Replace $f(x)$ with y:

$$y = \frac{4x+1}{3-x}$$

Solve for x:

$$y(3-x) = 4x+1$$

$$3y - xy = 4x+1$$

$$3y - 1 = 4x + xy$$

$$3y - 1 = x(4+y)$$

$$\frac{3y-1}{4+y} = x$$

Switch x and y:

$$y = \frac{3x-1}{4+x}$$

Replace y with $f^{-1}(x)$:

$$\boldsymbol{f^{-1}(x) = \frac{3x-1}{4+x}}$$

Check your answer:

$$(f \circ f^{-1})(x) = x$$

$$x = \frac{4\left(\frac{3x-1}{4+x}\right)+1}{3-\left(\frac{3x-1}{4+x}\right)}$$

$$x\left(3-\left(\frac{3x-1}{4+x}\right)\right) = 4\left(\frac{3x-1}{4+x}\right)+1$$

$$3x-\left(\frac{x(3x-1)}{4+x}\right) = \frac{4(3x-1)}{4+x}+1$$

$$3x-\left(\frac{3x^2-x}{4+x}\right) = \frac{12x-4}{4+x}+1$$

$$3x(4+x)-(3x^2-x) = 12x-4+1(4+x)$$

$$12x+3x^2-3x^2+x = 12x-4+4+x$$

$$13x = 13x$$

$$x = x$$

b.

$$(f \circ g)(x) = f\big(g(x)\big) = f(2x-6)$$

$$= \frac{4(2x-6)+1}{3-(2x-6)}$$

$$= \frac{8x-24+1}{3-2x+6}$$

$$= \boldsymbol{\frac{8x-23}{9-2x}}$$

c.

$$(g \circ f)(x) = g\big(f(x)\big) = g\left(\frac{4x+1}{3-x}\right)$$

$$= 2\left(\frac{4x+1}{3-x}\right) - 6$$

$$= \frac{2(4x+1)}{3-x} - 6$$

$$= \frac{8x+2}{3-x} - 6\left(\frac{3-x}{3-x}\right)$$

$$= \frac{8x+2}{3-x} - \frac{6(3-x)}{3-x}$$

$$= \frac{8x+2}{3-x} - \frac{18-6x}{3-x}$$

$$= \frac{8x+2-(18-6x)}{3-x}$$

$$= \frac{8x+2-18+6x}{3-x}$$

$$= \frac{14x-16}{3-x}$$

Problem #2

a.

$$f(2) = -1$$

$$g(2) = 2$$

$$4(-1) - 7(2) = -4 - 14 = \mathbf{-18}$$

b.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$\frac{f(-8)}{g(-8)} = \frac{9}{-3} = -\mathbf{3}$$

c.

$$(f \circ g)(x) = f\big(g(x)\big)$$

$$g(5) = -6$$

$$f(-6) = \mathbf{7}$$

d.

$$(f \circ f)(x) = f\big(f(x)\big)$$

$$f(0) = 1$$

$$f(1) = \mathbf{0}$$

e.

$$(fg)(x) = f(x) * g(x)$$

$$f(4) * g(4) = (-3)(-3) = \mathbf{9}$$

Problem #3

a. Any x-value can be plugged in:

$$D: (-\infty, \infty)$$

Regardless of the x-value, $f(x) > 0$:

$$R: (0, \infty)$$

b. Each time x increases by 1, the previous value of $f(x)$ is multiplied by a:

$$a = \frac{2}{5}$$

$$0 < a < 1$$

f is always decreasing

c. f is always decreasing but never reaches 0:

Horizontal asymptote at $y = 0$

No vertical asymptotes

d. When $x = 0$, $f(x) = C = 4$:

y-intercept: $(0, 4)$

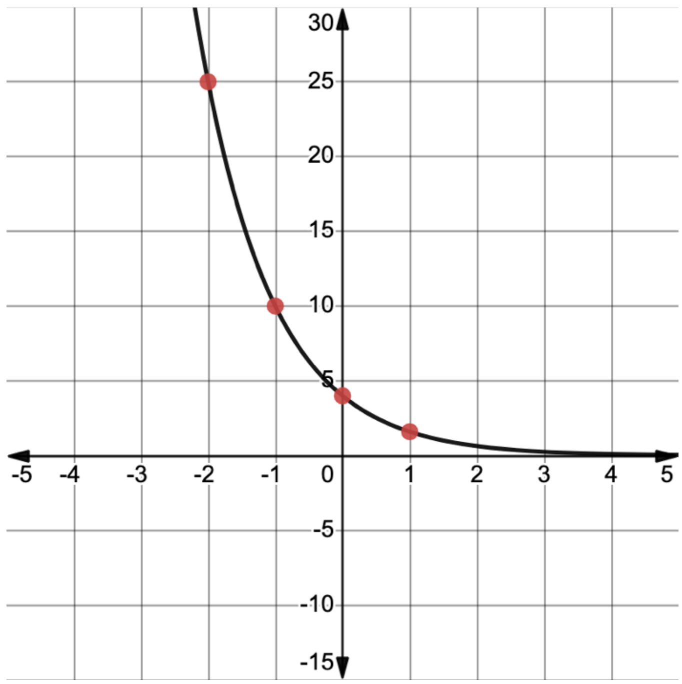
The graph of f never touches the x-axis, so there are no x-intercepts.

e. Yes, f is a one-to-one function because no two x-values produce the same y-value.

Yes, f has an inverse because it is a one-to-one function.

f.

x	$f(x)$
-2	$4\left(\frac{2}{5}\right)^{-2} = 4\left(\frac{5^2}{2}\right) = 4\left(\frac{25}{4}\right) = 25$
-1	$4\left(\frac{2}{5}\right)^{-1} = 4\left(\frac{5^1}{2}\right) = 4\left(\frac{5}{2}\right) = 10$
1	$4\left(\frac{2}{5}\right)^{1} = 4\left(\frac{2}{5}\right) = \frac{8}{5}$



Problem #4

a. Convert hours to minutes:

$$4 \text{ hours} * 60 \text{ minutes/hour} = 240 \text{ minutes}$$

Plug $t = 240$ into the function:

$$S(240) = 40 + \frac{1}{5}(240 - 35) = 40 + \frac{1}{5}(205) = 40 + 41 = \mathbf{81}$$

b. Yes – $S(t)$ is a linear function, which means it is a one-to-one function and has an inverse.

c. For the function S, the domain is the number of minutes a student studied and the range is the expected exam score.

For the inverse function S^{-1}, the domain is the expected exam score and the range is the number of minutes a student studied.

d. Solve the function for t:

$$S = 40 + \frac{1}{5}(t - 35)$$

$$S = 40 + \frac{1}{5}t - 7$$

$$S = 33 + \frac{1}{5}t$$

$$S - 33 = \frac{1}{5}t$$

$$5(S - 33) = t$$

$$5S - 165 = t$$

$$\boldsymbol{t = 5S - 165}$$

Plug 90 into the inverse function:

$$t = 5(90) - 165 = 450 - 165 = \mathbf{285}$$

To have an expected exam score of 90, a student must study for 285 minutes.

Problem #5

$$P = 10{,}000$$

$$r = .04$$

a. If interest is compounded annually, $n = 1$.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A(t) = 10{,}000\left(1 + \frac{.04}{1}\right)^{1t}$$

$$A(t) = 10{,}000(1.04)^{t}$$

b. If interest is compounded quarterly, $n = 4$.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A(t) = 10{,}000\left(1 + \frac{.04}{4}\right)^{4t}$$

$$A(t) = 10{,}000(1.01)^{4t}$$

c. If interest is compounded continuously, use the equation:

$$A = Pe^{rt}$$

$$A(t) = 10{,}000e^{.04t}$$

Problem #6

a. The half-life is $k = 8$ hours and the initial amount is $C = 10$ mg.

$$A(t) = C\left(\frac{1}{2}\right)^{\frac{t}{k}}$$

$$A(t) = 10\left(\frac{1}{2}\right)^{\frac{t}{8}}$$

b. Plug $t = 4$ into the function:

$$A(4) = 10\left(\frac{1}{2}\right)^{\frac{4}{8}} = 10\left(\frac{1}{2}\right)^{\frac{1}{2}} = 10\sqrt{\frac{1}{2}} \approx \mathbf{7.1\ mg}$$

A person has about 7.1 mg of Adderall in their system 4 hours after taking a 10 mg dosage.

c. It takes 8 hours for the amount to reach half of its previous value:

$$10\left(\tfrac{1}{2}\right) = 5 \text{ mg remaining after 8 hours}$$

$$5\left(\tfrac{1}{2}\right) = 2.5 \text{ mg remaining after 16 hours}$$

$$2.5\left(\tfrac{1}{2}\right) = 1.25 \text{ mg remaining after } \textbf{24 hours}$$

Problem #7

a.

$$-2\ln 4x = -6$$

$$\ln 4x = 3$$

$$e^{\ln 4x} = e^3$$

$$4x = e^3$$

$$x = \frac{1}{4}e^3$$

b.

$$\log 2x = 4$$

$$10^{\log 2x} = 10^4$$

$$2x = 10{,}000$$

$$x = 5{,}000$$

c.

$$3\log_2(9x) = -6$$

$$\log_2(9x) = -2$$

$$2^{\log_2(9x)} = 2^{-2}$$

$$9x = \frac{1}{2^2}$$

$$9x = \frac{1}{4}$$

$$x = \frac{1}{36}$$

d.

$$3(2^x) = 24$$

$$2^x = 8$$

$$x = 3$$

e.

$$e^{2x} = e^{\frac{1}{3}}$$

$$\ln e^{2x} = \ln e^{\frac{1}{3}}$$

$$2x = \frac{1}{3}$$

$$x = \frac{1}{6}$$

Problem #8

a. You can only find the logarithm of positive values.

$$3x - 12 > 0$$

$$3x > 12$$

$$x > 4$$

$$(4, \infty)$$

b. You can only find the logarithm of positive values.

$$x^2 - 25 > 0$$

$$x^2 > 25$$

$$x > 5 \qquad\qquad x < -5$$

$$(-\infty, -5) \cup (5, \infty)$$

c. You can only find the logarithm of positive values, **and** you can only take the square root of values greater than or equal to zero.

$$5 - \sqrt{x+3} > 0 \qquad \text{and} \qquad x + 3 \geq 0$$

$$-\sqrt{x+3} > -5 \qquad\qquad x \geq -3$$

$$\sqrt{x+3} < 5$$

$$x + 3 < 25$$

$$x < 22$$

$$[-3, 22)$$

d. You can only find the logarithm of positive values, but exponential functions are always positive.

$$1^x > 0$$

$$(-\infty, \infty)$$