MATH 021 – Final (New Material) – Practice Exam Solutions

Problem #1

a. $(-\infty, -1) \cup (4, \infty)$

Find the interval(s) of x-values for which the graph is below the x-axis. The endpoints are not included in the solution set because they are on the x-axis, not below it.

b. $(-\infty, -1] \cup [4, \infty)$

Find the interval(s) of x-values for which the graph is at or below the x-axis. The endpoints are included in the solution set because they are on the x-axis.

c. $(-1, 4)$

Find the interval(s) of x-values for which the graph is above the x-axis. The endpoints are not included in the solution set because they are on the x-axis, not above it.

d. $[-1, 4]$

Find the interval(s) of x-values for which the graph is at or above the x-axis. The endpoints are included in the solution set because they are on the x-axis.

Problem #2

a.

Rewrite the inequality so that one side is equal to 0:

$$x^2 - 2x - 9 > 0$$

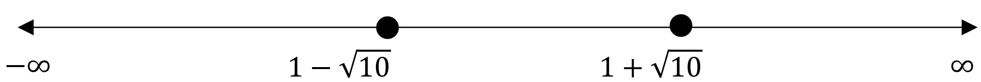
Solve as an equality to find the boundary numbers:

$$x^2 - 2x - 9 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-9)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{40}}{2} = \frac{2 \pm 2\sqrt{10}}{2} = 1 \pm \sqrt{10}$$

Use the boundary numbers to separate the number line into disjoint intervals:



Make a table of test values to see whether each interval is positive or negative:

x	$f(x)$	**Positive/Negative**
-3	$(-3)^2 - 2(-3) - 9 = 6$	Positive
0	$(0)^2 - 2(0) - 9 = -9$	Negative
5	$(5)^2 - 2(5) - 9 = 6$	Positive

Find the intervals where $x^2 - 2x - 9 > 0$ and write in interval notation:

$$(-\infty, 1 - \sqrt{10}) \cup (1 + \sqrt{10}, \infty)$$

b.

Rewrite the inequality so that one side is equal to 0:

$$-x^2 + 6x - 9 < 0$$

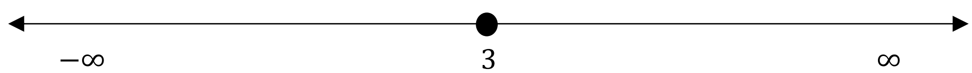
Solve as an equality to find the boundary numbers:

$$-x^2 + 6x - 9 < 0$$

$$(x - 3)(-x + 3) < 0$$

$$x = 3$$

Use the boundary number to separate the number line into disjoint intervals:



Make a table of test values to see whether each interval is positive or negative:

$\boldsymbol{x}$	$\boldsymbol{f(x)}$	**Positive/Negative**
0	$-(0)^2 + 6(0) - 9 = -9$	Negative
4	$-(4)^2 + 6(4) - 9 = -1$	Negative

Find the intervals where $-x^2 + 6x - 9 < 0$ and write in interval notation:

$$(-\infty, 3) \cup (3, \infty)$$

c.

Rewrite the inequality so that one side is equal to 0:

$$-2x^2 + x - 3 \leq 0$$

Solve as an equality to find the boundary numbers:

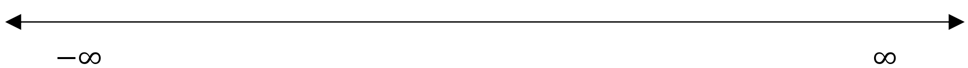
$$-2x^2 + x - 3 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(-2)(-3)}}{2(-2)}$$

$$x = \frac{-1 \pm \sqrt{-23}}{-4}$$

There are no real solutions for x.

There are no boundary numbers, so the entire number line is one interval:



Test a value to see whether the function is entirely positive or entirely negative:

x	$f(x)$	**Positive/Negative**
0	$-2(0)^2 + 0 - 3 = -3$	Negative

Because the function is entirely negative, $-2x^2 + x - 3 \leq 0$ for all real numbers:

$$(-\infty, \infty)$$

Problem #3

Find an equation for the area of the rectangle:

$$A = LW$$

$$A = (W + 6)(W)$$

$$A = W^2 + 6W$$

Set up an inequality to show that the area is 216 square inches or less:

$$W^2 + 6W \leq 216$$

Solve as an equality to find the maximum width:

$$W^2 + 6W - 216 \leq 0$$

$$W^2 + 6W - 216 \leq 0$$

$$(W + 18)(W - 12) \leq 0$$

$$W = -18 \qquad \text{or} \qquad W = 12$$

The width of a rectangle can't be negative, so $W = 12$.

Add 6 to find the maximum length:

$$L = W + 6 = 12 + 6 = 18$$

The length of the rectangle is 18 inches or less.

$$0 < L \leq 18$$

Problem #4

Set up an inequality:

$$-5x^2 + 30x + 3 \geq 28$$

Rewrite the inequality so that one side is equal to 0:

$$-5x^2 + 30x - 25 \geq 0$$

Solve as an equality to find the boundary numbers:

$$-5x^2 + 30x - 25 = 0$$

$$-5(x^2 - 6x + 5) = 0$$

$$-5(x - 1)(x - 5) = 0$$

$$x = 1 \qquad\qquad x = 5$$

We know that the height of the ball would increase and then decrease, meaning that the parabola opens downward (this can be verified by the negative leading coefficient). That means that the height is greater than 28 feet between the two boundary numbers.

The ball is at least 28 feet above the ground between 1 second and 5 seconds after it hits the bat.

Problem #5

a.

Parent graph: $f(x) = x^2$

1. **Shift left 2 units:** Plug $(x+2)$ in for x.
2. **Reflect across x-axis:** Put a negative sign outside the parenthesis.
3. **Shift upward 1 unit:** Add 1 to the function.

$$f(x) = -(x+2)^2 + 1$$

b.

Parent graph: $f(x) = |x|$

1. **Shift left 1 unit:** Plug $(x+1)$ in for x.
2. **Shift downward 4 units:** Subtract 4 from the function.

$$f(x) = |x+1| - 4$$

c.

Parent graph: $f(x) = \sqrt{x}$

1. **Shift left 4 unit:** Plug $(x+4)$ in for x.
2. **Shift downward 3 units:** Subtract 3 from the function.

$$f(x) = \sqrt{x+4} - 3$$

d.

Parent graph: $f(x) = \sqrt{x}$

1. **Reflect across x-axis:** Put a negative sign outside the radical.
2. **Reflect across y-axis:** Make x negative.

$$f(x) = -\sqrt{-x}$$

Problem #6

a. **Shift right 5 units:** Plug $(x-5)$ in for x.
Shift downward 10 units: Subtract 10 from the function.

$$g(x) = 6(x-5)^2 + 2(x-5) - 4 - 10$$

$$g(x) = 6(x^2 - 10x + 25) + 2x - 10 - 14$$

$$g(x) = 6x^2 - 60x + 150 + 2x - 24$$

$$g(x) = 6x^2 - 58x + 126$$

b. **Reflect across y-axis:** Make x negative.

$$g(x) = 6(-x)^2 + 2(-x) - 4$$

$$g(x) = 6x^2 - 2x - 4$$

c. **Vertically shrink by a factor of ½:** Multiply the entire function by ½.

$$g(x) = \frac{1}{2}(6x^2 + 2x - 4)$$

$$g(x) = 3x^2 + x - 2$$

Problem #7

a. **Shift left 100 units:** Plug $(x+100)$ in for x.
Shift upward 400 units: Add 400 to the function.

$$g(x) = \sqrt{x+100} + 400$$

b. **Reflect across x-axis:** Put a negative sign outside the radical.

$$g(x) = -\sqrt{x}$$

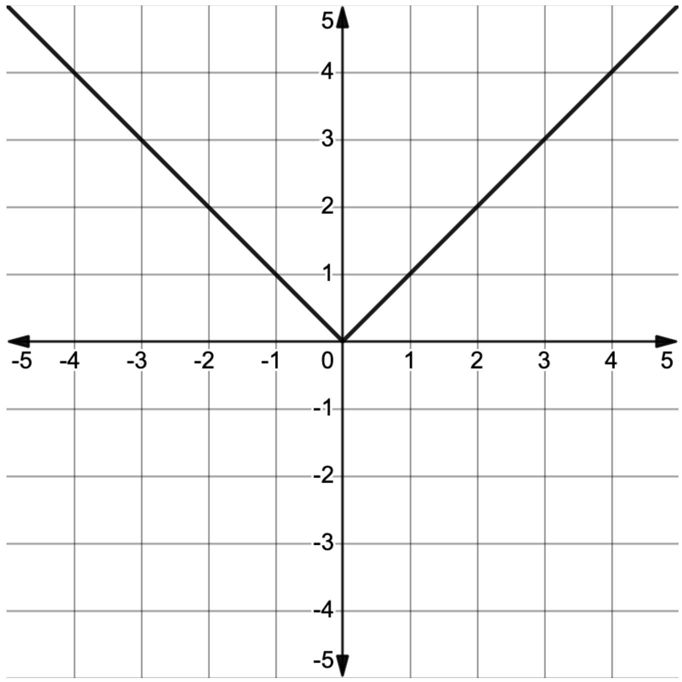
c. **Horizontally stretch by a factor of 2:** Plug $\frac{1}{2}x$ in for x.

$$g(x) = \sqrt{\frac{1}{2}x}$$

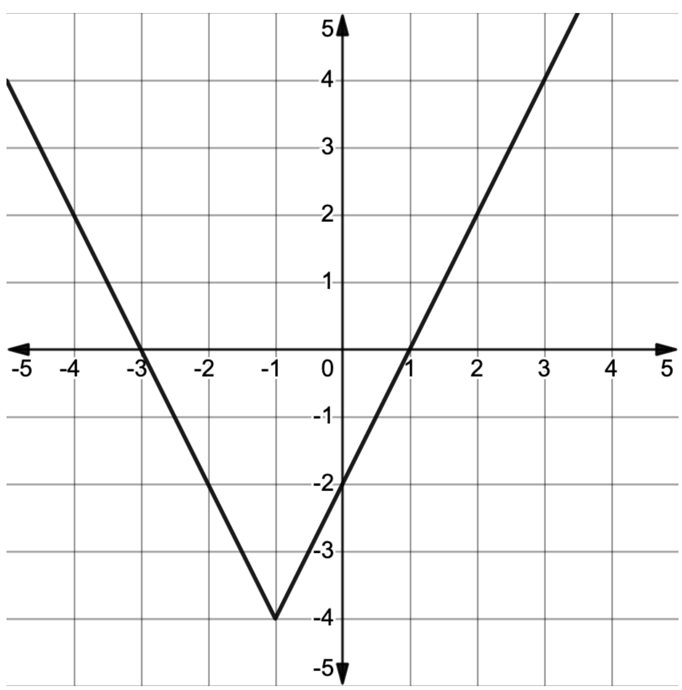
Problem #8

a.

Parent graph: $f(x) = |x|$

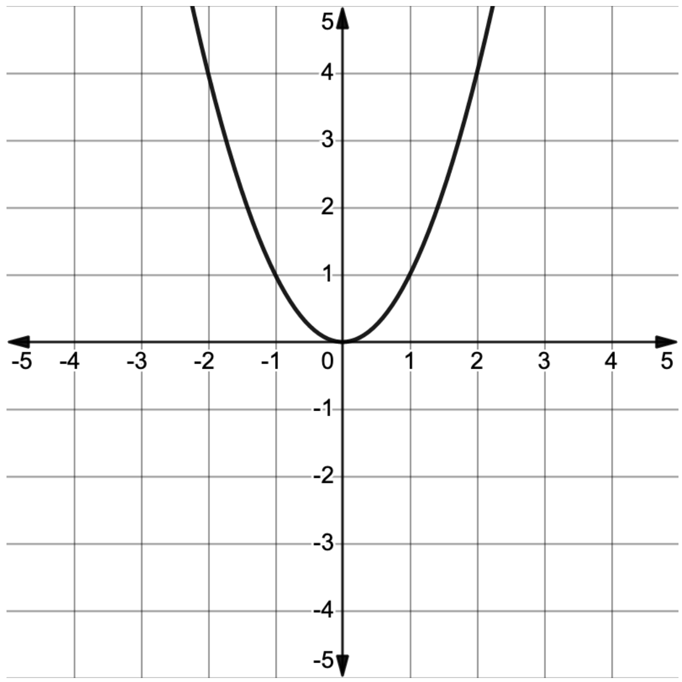


1. **Shift left 1 unit:** $(x+1)$ is plugged in for x.
2. **Vertically stretch by a factor of 2:** $|x+1|$ is multiplied by 2.
3. **Shift downward 4 units:** 4 is subtracted from the function.



b.

Parent graph: $f(x) = x^2$



1. **Horizontally shrink by a factor of ½:** $2x$ is plugged in for x, which means we need to divide each x-value from the parent graph by 2.
2. **Reflect across x-axis:** There is a negative sign outside the parentheses.