MATH 021 – Exam 3 – Practice Exam Solutions

Problem #1

a. The leading coefficient is negative because the parabola opens downward.

b. The discriminant is positive because there are two real solutions for $f(x) = 0$ (the graph intersects the x-axis twice).

c. H – $f(x) = -2(x+1)^2 + 6$

The leading coefficient must be negative because the parabola opens downward. This eliminates A, B, C, and D. Based on the remaining answer choices, we can infer that the leading coefficient is -2.

The vertex is at $(-1, 6)$, and the formula is written in vertex form:

$$f(x) = a(x-h)^2 + k$$
$$f(x) = -2(x+1)^2 + 6$$

Problem #2

a. The leading coefficient is positive because the parabola opens upward.

b. The discriminant is negative because there are no real solutions for $f(x) = 0$ (the graph doesn't intersect the x-axis).

c. C – $f(x) = (x-3)^2 + 1$

The leading coefficient must be positive because the parabola opens upward. This eliminates E, F, G, and H. Based on the remaining answer choices, we can infer that the leading coefficient is 1, which means that it doesn't need to be included.

The vertex is at $(3, 1)$, and the formula is written in vertex form:

$$f(x) = a(x-h)^2 + k$$
$$f(x) = (x-3)^2 + 1$$

Problem #3

a. Plug $a = -4$ and $b = -24$ into the vertex formula:

$$h = -\frac{b}{2a} = -\frac{-24}{2(-4)} = \frac{24}{-8} = -3$$

$$k = f(-3) = -4(-3)^2 - 24(-3) + 1 = -4(9) + 72 + 1 = 37$$

Plug $h = -3$ and $k = 37$ into vertex form:

$$f(x) = -4(x+3) + 37$$

b. Replace $f(x)$ with y:

$$y = -4x^2 - 24x + 1$$

Divide by -4 to cancel out the coefficient of x^2:

$$-\frac{1}{4}y = x^2 + 6x - \frac{1}{4}$$

Isolate the x terms:

$$-\frac{1}{4}y + \frac{1}{4} = x^2 + 6x$$

Complete the square:

$$-\frac{1}{4}y + \frac{1}{4} + \left(\frac{6}{2}\right)^2 = x^2 + 6x + \left(\frac{6}{2}\right)^2$$

$$-\frac{1}{4}y + \frac{1}{4} + 9 = x^2 + 6x + 9$$

$$-\frac{1}{4}y + \frac{37}{4} = x^2 + 6x + 9$$

Factor the perfect square:

$$-\frac{1}{4}y + \frac{37}{4} = (x+3)^2$$

Solve for y:

$$-\frac{1}{4}y = (x+3)^2 - \frac{37}{4}$$

$$y = -4(x+3)^2 + 37$$

Replace y with $f(x)$:

$$f(x) = -4(x+3)^2 + 37$$

Problem #4

a. Plug $x = 1$ into $P(x)$:

$$P(1) = 32(1) - 4(1)^2 - 15 = 32 - 4 - 15 = 13$$

When the company produces 1 million units, the total profit is $13 million.

b. Use the vertex formula to find the x-coordinate of the vertex, which is the number of units where $P(x)$ is maximized:

$$-\frac{b}{2a} = -\frac{32}{2(-4)} = -\frac{32}{-8} = 4 \text{ million units}$$

c. Find $P(4)$ to find the y-coordinate of the vertex, which is the maximum profit:

$$P(4) = 32(4) - 4(4)^2 - 15 = 128 - 64 - 15 = \$49 \text{ million}$$

d. The profit is increasing until it reaches the vertex, and then it is decreasing.

Because it's not possible to produce a negative number of units, the domain starts at $x = 0$.

Increasing: $(0,4)$

Decreasing: $(4, \infty)$

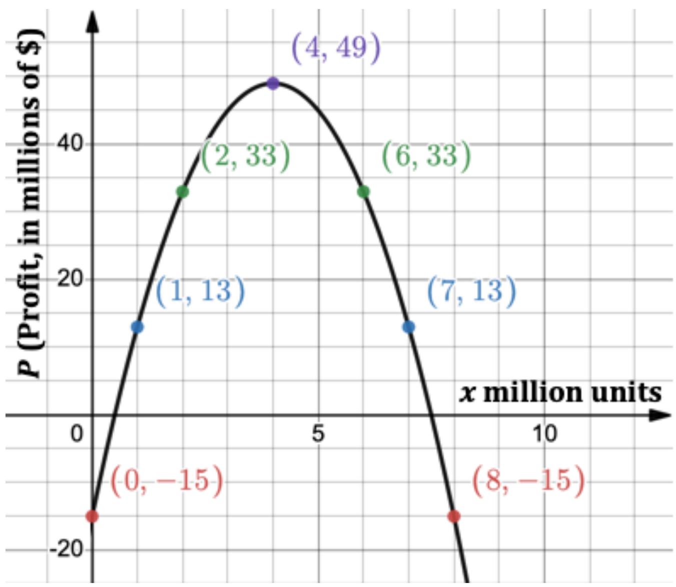
e. Graph features:

- Parabola opens downward because the leading coefficient is negative: $a = -4$
- Vertex (found in parts b and c above): $(4, 49)$
- Axis of symmetry (vertical line through the vertex): $x = 4$
- $P(1) = 13$ (found in part a above)

Table of values:

x	$P(x)$
0	$32(0) - 4(0)^2 - 15 = -15$
2	$32(2) - 4(2)^2 - 15 = 33$

The graph can be reflected along the axis of symmetry at $x = 4$, so we do not have to calculate values for the right side of the parabola.



Problem #5

a. 2 real solutions

$$-3x^2 + 2x + 6 = 0$$

$$discriminant = \ 2^2 - 4(-3)(6) = 76$$

There are <u>two</u> real solutions because the discriminant is positive.

b. 1 real solution

$$x^2 - 6x + 9 = 0$$

$$discriminant = \ (-6)^2 - 4(1)(9) = 0$$

There is <u>one</u> real solution because the discriminant is zero.

c. No real solutions

$$4x^2 - 3x + 5 = 0$$

$$discriminant = \ (-3)^2 - 4(4)(5) = -71$$

There are <u>no</u> real solutions because the discriminant is negative.

Problem #6

$f(x) = x^2 + 2x - 4$

$f(x+h) = (x+h)^2 + 2(x+h) - 4$
$f(x+h) = x^2 + 2xh + h^2 + 2x + 2h - 4$

$$Difference\ quotient = \frac{x^2 + 2xh + h^2 + 2x + 2h - 4 - (x^2 + 2x - 4)}{h}$$

$$= \frac{x^2 + 2xh + h^2 + 2x + 2h - 4 - x^2 - 2x + 4}{h}$$

$$= \frac{2xh + h^2 + 2h}{h}$$

$$= 2x + h + 2$$

Problem #7

a. $p(1) = 1^2 + 5(1) = \ 6$

1 day after a rumor begins, approximately 6% of the town has heard some version of the rumor.

b. $p(t) = t^2 + 5t$

$p(t+h) = (t+h)^2 + 5(t+h)$
$p(t+h) = t^2 + 2th + h^2 + 5t + 5h$

$$Difference\ quotient = \frac{t^2 + 2th + h^2 + 5t + 5h - (t^2 + 5t)}{h}$$

$$= \frac{t^2 + 2th + h^2 + 5t + 5h - t^2 - 5t}{h}$$

$$= \frac{2th + h^2 + 5h}{h}$$

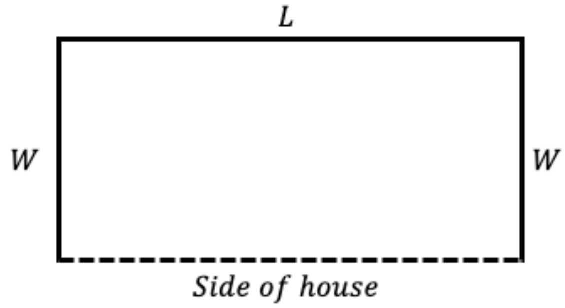
$$= 2t + h + 5$$

c. $2(2) + (0.1) + 5 = 9.1$

During the time period from 2 days to 2.1 days after the rumor begins, the percentage of the town that has heard the rumor increases by an average of approximately 9.1% per day.

Problem #8

a. Start by drawing a sketch:



200 feet of fencing will be used to enclose three sides of the rectangle:

$$200 = 2W + L$$

Solve the equation for L to put all three sides in terms of W:

$$L = 200 - 2W$$

Write an equation for the area of the fence:

$$A = LW$$

$$A = (200 - 2W)(W)$$

$$A = 200W - 2W^2$$

Use the vertex formula to find the width at which the area is maximized:

$$-\frac{b}{2a} = -\frac{200}{2(-2)} = 50$$

$$W = 50 \text{ ft}$$

$$L = 200 - 2(50) = 100 \text{ ft}$$

The dimensions that maximized the fenced-in are 50 ft x 100 ft.

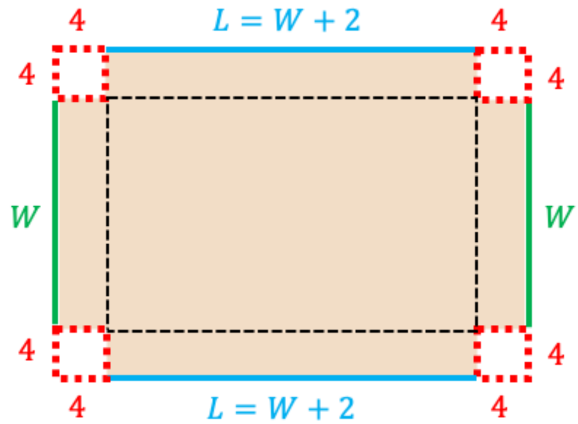
b. Plug the dimensions into the area equation:

$$A = LW$$

$$A = (50\text{ft})(100\text{ft}) = 5{,}000 \text{ ft}^2$$

Problem #9

Start by drawing a sketch:



Write an equation for the volume of the box:

$$V = LWH = 192$$

$$192 = (W+2)(W)(4)$$

Divide both sides by 4:

$$48 = (W+2)(W)$$

Distribute:

$$48 = W^2 + 2W$$

Complete the square:

$$\left(\frac{2}{2}\right)^2 + 48 = W^2 + 2W + \left(\frac{2}{2}\right)^2$$

$$1 + 48 = W^2 + 2W + 1$$

Factor:

$$49 = (W+1)^2$$

Square root property:

$$\pm\sqrt{49} = W + 1$$

$$7 = W + 1 \quad \text{or} \quad -7 = W + 1$$
$$6 = W \qquad\qquad -8 = W$$

Width can't be negative:

$$W = 6$$

Use the width to find the length:

$$L = 6 + 2 = 8$$

Dimensions of <u>box</u>:

$$L = 8 \text{ in.}$$
$$W = 6 \text{ in.}$$
$$H = 4 \text{ in.}$$

Dimensions of <u>piece of cardboard</u>:

$$L = 4 + 8 + 4 = \mathbf{16\ in.}$$
$$W = 4 + 6 + 4 = \mathbf{14\ in.}$$

Problem #10

a. Find the point(s) where $f(x) = 0$.

$$x = -1$$

b. Because f has one zero, the discriminant is equal to 0.

c. x-intercept: $(-1, 0)$

y-intercept: $(0, 2)$

d. Positive (above the x-axis) for all values except $x = -1$, where $f(x) = 0$:

$$(-\infty, -1) \cup (-1, \infty)$$

There are no intervals where the function is negative.

e. Increasing (sloping upward from left to right):

$$(-1, \infty)$$

Decreasing (sloping downward from left to right):

$$(-\infty, -1)$$

f. The function is increasing from $x = -1 \rightarrow x = 0$, so the average rate of change is positive.

g. Find the slope of the straight line connecting the point where $x = -1$ and the point where $x = 0$:

Points: $(-1, 0)$ and $(0, 2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{0 - (-1)} = \frac{2}{1} = 2$$

Problem #11

a.

$$3i(3-2i)(3-2i)$$

$$3i(9-12i+4i^2)$$

$$3i(9-12i+4(-1))$$

$$3i(9-12i-4)$$

$$3i(5-12i)$$

$$15i-36i^2$$

$$15i-36(-1)$$

$$\mathbf{36+15}\boldsymbol{i}$$

b.

$$2i+5-21+12i$$

$$\mathbf{-16+14}\boldsymbol{i}$$

c.

$$\frac{3+i}{6-3i}\times\frac{6+3i}{6+3i}$$

$$\frac{18+9i+6i+3i^2}{36+18i-18i-9i^2}$$

$$\frac{18+15i+3(-1)}{36-9(-1)}$$

$$\frac{15+15i}{45}$$

$$\frac{15}{45}+\frac{15}{45}i$$

$$\mathbf{\frac{1}{3}+\frac{1}{3}}\boldsymbol{i}$$

d.

$$i^{11} - i^4 - 4i^3$$

$$i(i^2)^5 - (i^2)^2 - 4i(i^2)$$

$$i(-1)^5 - (-1)^2 - 4i(-1)$$

$$i(-1) - (1) + 4i$$

$$-i - 1 + 4i$$

$$\mathbf{-1 + 3}\boldsymbol{i}$$

<u>Problem #12</u>

a.

$$-x^2 - 4x + 6 = 0$$

$$discriminant = (-4)^2 - 4(-1)(6) = 16 + 24 = 40$$

b. The solutions are <u>real</u> because the discriminant is not negative.

c. There are <u>two</u> solutions because the discriminant is not zero.

d. Solve using the quadratic formula (remember that we calculated the discriminant in part a):

$$x = \frac{-(-4) \pm \sqrt{40}}{2(-1)}$$

$$x = \frac{4 \pm 2\sqrt{10}}{-2}$$

$$x = -2 \pm \sqrt{10}$$

$$x = -2 - \sqrt{10} \qquad\qquad x = -2 + \sqrt{10}$$

Problem #13

a.

$$2x^2 + 4x + 10 = 0$$

$$discriminant = 4^2 - 4(2)(10) = 16 - 80 = -64$$

b. The solutions are <u>nonreal complex</u> because the discriminant is negative.

c. There are <u>two</u> solutions because the discriminant is not zero.

d. Solve using the quadratic formula (remember that we calculated the discriminant in part a):

$$x = \frac{-4 \pm \sqrt{-64}}{2(2)}$$

$$x = \frac{-4 \pm i\sqrt{64}}{4}$$

$$x = \frac{-4 \pm 8i}{4}$$

$$x = -1 \pm 2i$$

$$x = -1 - 2i \qquad x = -1 + 2i$$