MATH140 Exam 2 – Sample Test Solutions

Problem 1

Answer: Domain of $f(x)$: $(\mathbf{1}, \mathbf{5}) \cup (\mathbf{5}, \infty)$

Denominator:

$(x^2 - 25)\sqrt{x-1} \neq 0$

$(x^2 - 25) \neq 0, \sqrt{x-1} \neq 0$

$x^2 \neq 25, x - 1 \neq 0$

$x \neq -5, 5, 1$

Square Root:

$x - 1 \geq 0$

$x \geq 1$

Answer: Domain of $g(x)$: $\left(-\infty, -\frac{5}{2}\right) \cup (\mathbf{1}, \infty)$

Logarithm:

$|4x + 3| - 7 > 0$

$|4x + 3| > 7$

$4x + 3 > 7$ OR $4x + 3 < -7$

$4x > 4$ OR $4x < -10$

$x > 1$ OR $x < -5/2$

Problem 2

$$\boldsymbol{Answer}: \boldsymbol{V(t)} = \begin{cases} \mathbf{70,000 - 6000}\boldsymbol{t}, & \boldsymbol{t} \leq \mathbf{5} \\ \mathbf{40,000(.93)}^{\boldsymbol{t}-\mathbf{5}}, & \boldsymbol{t} > \mathbf{5} \end{cases}$$

First 5 years:
Linear Function
$b = 70{,}0000$
$m = -6{,}000$

After 5 years:
Exponential Decay
$A = 70{,}000 - 6{,}000(5) = 40{,}000$
$b = 1 - .07 = 0.93$

Problem 3

$$\boldsymbol{Answer}: \boldsymbol{t} = \frac{\mathbf{25\,ln(0.2)}}{\mathbf{ln\,(0.5)}}$$

$$f(t) = A\left(\frac{1}{2}\right)^{t/25}$$

$$.2A = A\left(\frac{1}{2}\right)^{\frac{t}{25}}$$

$$0.2 = (0.5)^{\frac{t}{25}}$$

$$\ln(0.2) = \frac{t}{25}\ln(0.5)$$

$$t = \frac{25\ln(0.2)}{\ln\,(0.5)}$$

Problem 4

Answer: $\boldsymbol{D(t) = 4 + 2\cos\left(\frac{\pi}{6}(t-7)\right)}$

$$M = \frac{2+6}{2} = 4$$

$$A = \frac{6-2}{2} = 2$$

$$P = 12$$

$$C = 7$$

Use the cosine function since you are given a data point about when high-tide (the maximum) occurs.

$$D(t) = M + A\cos\left(\frac{2\pi}{P}(t-c)\right)$$

$$D(t) = 4 + 2\cos\left(\frac{2\pi}{12}(t-7)\right)$$

Problem 5

Answer: $\boldsymbol{x = 2 + \frac{1}{4}\cos^{-1}\left(\frac{2}{3}\right) + \frac{\pi}{2}n \quad and \quad x = 2 - \frac{1}{4}\left(\cos^{-1}\left(\frac{2}{3}\right)\right) + \frac{\pi}{2}n}$

$$\cos(4x-8) = \frac{2}{3}$$

$$4x - 8 = \cos^{-1}\left(\frac{2}{3}\right) \quad and \quad 4x - 8 = -\left(\cos^{-1}\left(\frac{2}{3}\right)\right)$$

$$4x - 8 = \cos^{-1}\left(\frac{2}{3}\right) + 2\pi n \quad and \quad 4x - 8 = -\left(\cos^{-1}\left(\frac{2}{3}\right)\right) + 2\pi n$$

$$4x = 8 + \cos^{-1}\left(\frac{2}{3}\right) + 2\pi n \quad and \quad 4x = 8 - \left(\cos^{-1}\left(\frac{2}{3}\right)\right) + 2\pi n$$

$$x = \frac{8}{4} + \frac{1}{4}\cos^{-1}\left(\frac{2}{3}\right) + \frac{2\pi n}{4} \quad and \quad x = \frac{8}{4} - \frac{1}{4}\left(\cos^{-1}\left(\frac{2}{3}\right)\right) + \frac{2\pi n}{4}$$

$$x = 2 + \frac{1}{4}\cos^{-1}\left(\frac{2}{3}\right) + \frac{\pi}{2}n \quad and \quad x = 2 - \frac{1}{4}\left(\cos^{-1}\left(\frac{2}{3}\right)\right) + \frac{\pi}{2}n$$

Problem 6

Answer: $\mathbf{17}\ \boldsymbol{ft/s}$

$$\frac{s(t_2) - s(t_1)}{t_2 - t_1} = \frac{s(5) - s(2)}{5 - 2}$$

$$\frac{(3(5)^2 - 5) - (3(2)^2 - 2)}{5 - 2} = \frac{70 - 10}{5 - 2} = \frac{60}{3} = 20\ ft/s$$

Problem 7

Answer: $-\infty$

When you plug in you get 0/0, so simplify by factoring

$$\lim_{x \to 5^+} \frac{25 - x^2}{(5 - x)^2} = \lim_{x \to 5^+} \frac{(5 - x)(5 + x)}{(5 - x)(5 - x)} = \lim_{x \to 5^+} \frac{(5 + x)}{(5 - x)}$$

Now when you plug in you get #/0, so test a value on the right side of 5 to see if the limit approaches positive infinity or negative infinity…

$$\text{Try } 5.1 \rightarrow \ \frac{5.1 + 5}{5 - 5.1} = \frac{+}{-} = -\infty$$

Problem 8

Answer: **7/4**

When you plug in you get 0/0, so simplify by factoring

$$\lim_{x \to 3} \frac{2x^2 - 5x - 3}{x^2 - x - 6} = \lim_{x \to 3} \frac{(2x + 1)(x - 3)}{(x - 3)(x + 2)} = \lim_{x \to 3} \frac{(2x + 1)}{(x + 2)}$$

Now when you plug in you get…

$$\frac{2(3) + 1}{3 + 2} = \frac{7}{5}$$

Problem 9

Answer: $-$ **14**

Since this is an absolute value limit, start by determining if what's inside the absolute value will be positive or negative as $x \to 7$ from the right.

$|7 - x|$ will be negative as $x \to 7$ from the right, so to remove the absolute value signs, we must insert a negative…

$$\lim_{x \to 7^+} \frac{14x - 2x^2}{-(7 - x)} = \lim_{x \to 7^+} \frac{2x(7 - x)}{-(7 - x)} = \lim_{x \to 7^+} -2x = -14$$

Problem 10

Answer: 0

Use the Squeeze Theorem…

$$-1 \le \sin\left(\frac{2}{x}\right) \le 1$$

$$2 \le 3 + \sin\left(\frac{2}{x}\right) \le 5$$

$$3x^2 \le x^2\left(3 + \sin\left(\frac{2}{x}\right)\right) \le 5x^2$$

$$\lim_{x \to 0} 3x^2 \le \lim_{x \to 0}\left[x^2\left(3 + \sin\left(\frac{2}{x}\right)\right)\right] \le \lim_{x \to 0} 5x^2$$

$$0 \le \lim_{x \to 0}\left[x^2\left(3 + \sin\left(\frac{2}{x}\right)\right)\right] \le 0$$

$$\lim_{x \to 0}\left[x^2\left(3 + \sin\left(\frac{2}{x}\right)\right)\right] = 0$$

Problem 11

Answer: 1

When you evaluate the limit you get:

$$\frac{\lim_{x\to 3}(f(x)-1)}{0} = 5$$

Since this limit evaluates to a number, that means it must be a 0/0 limit. So the top of the fraction above must equal 0.

$$\lim_{x\to 3}(f(x)-1) = 0$$

$$\lim_{x\to 3} f(x) = 1$$

Problem 12

Answer: 0

When you evaluate the limit you get:

$$\frac{5}{\lim_{x\to 2} g(x)} = \infty$$

Since this limit evaluates to ∞, that means it must be a #/0 limit. So the bottom of the fraction must equal 0.

$$\lim_{x\to 2} g(x) = 0$$

Problem 13

$f(20) = f(15) + 10$

The population of Happy Valley in 2020 is 10,000 people more than the population of Happy Valley in 2015.

$f^{-1}(2) = 5$

The population of Happy Valley in 2005 is 17,000 people.

Problem 14

$-7, 1$