PHYS 250 Exam 1 – Supplement

Circular Motion

Centripetal Acceleration

- There are two ways of defining acceleration we need to be aware of.
- The one we've been using so far deals with linear motion and is typically used when an object's speed changes over a given period of time.
- However, there is another form of acceleration we must consider that results in **circular motion** due to a **change in the direction of velocity**.
- For circular motion, an object can have **an acceleration that causes its direction to constantly change while still maintaining a constant speed**.
- This is referred to as **centripetal acceleration**, and is defined by:

$$a_c = \frac{v^2}{r}$$

where ***v*** is the object's **velocity**, and ***r*** is the **radius** of the circular path.

- The **centripetal acceleration**, like all accelerations, is a **vector**, and it always **points towards the center of the circular path**.
- This means that when an object moves in a circular path, it's **velocity will always be tangent to the path**, while its **centripetal acceleration will always be perpendicular to the velocity**, and thus point towards the center.
- As the object moves along the circumference of its circular path, its **velocity and centripetal acceleration will continuously change direction**, however they will always remain perpendicular to one another.

Problem 1:

Two cats run in circles around a neighbor's dog, forcing it to stay put. As the two cats move in a circular path, Cat #1 is a distance *R* from the dog and maintains a constant speed *v*, while Cat #2 is at a of distance 4R from the dog and maintains a constant speed of 4*v*. Given this information, which of the statements below is/are true?

i. Both cats have the same (non-zero) centripetal acceleration.
ii. Both cats have zero acceleration because their speed is constant.
iii. Cat #2 has a centripetal acceleration that is four times that of Cat #1.
iv. Cat #2 completes each circle in one fourth the amount of time it takes Cat #1.
v. Both cats complete each circle in same amount of time.

A. i. and iv.
B. ii. and v.
C. iii. and v.
D. ii. and iv.
E. iii. and iv.

Centripetal Force

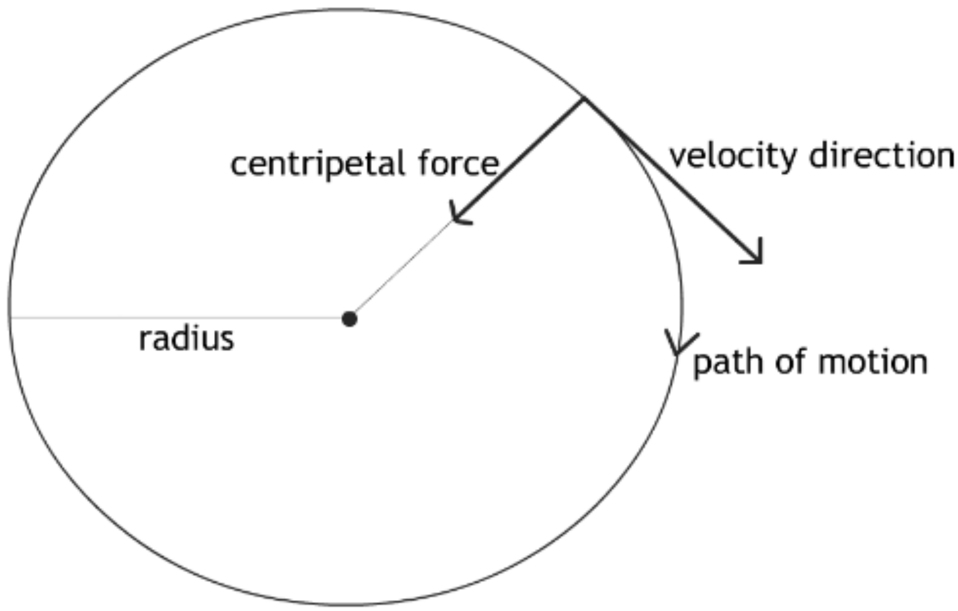
- We have now established that whenever an object is moving in a **circular path** there must be a centripetal acceleration acting on it.
- According to Newton's Second Law, $F_{Net} = ma$, **whenever there is an acceleration there must also be a net force pointing in the same direction** as the acceleration, and so we can assume that there must be one or more **forces that point towards the center**.
- It is this force (or combination of several forces) that causes the object's centripetal acceleration.
- **This force can be any of the forces we have previously encountered** (weight, normal force, friction, tension etc.), and is referred to as the **centripetal force, F_c**.
- Whether the centripetal force is just one force or the sum of several different forces all pointing towards the center (along the radial axis) it can be defined as:

$$F_c = m\frac{v^2}{r}$$

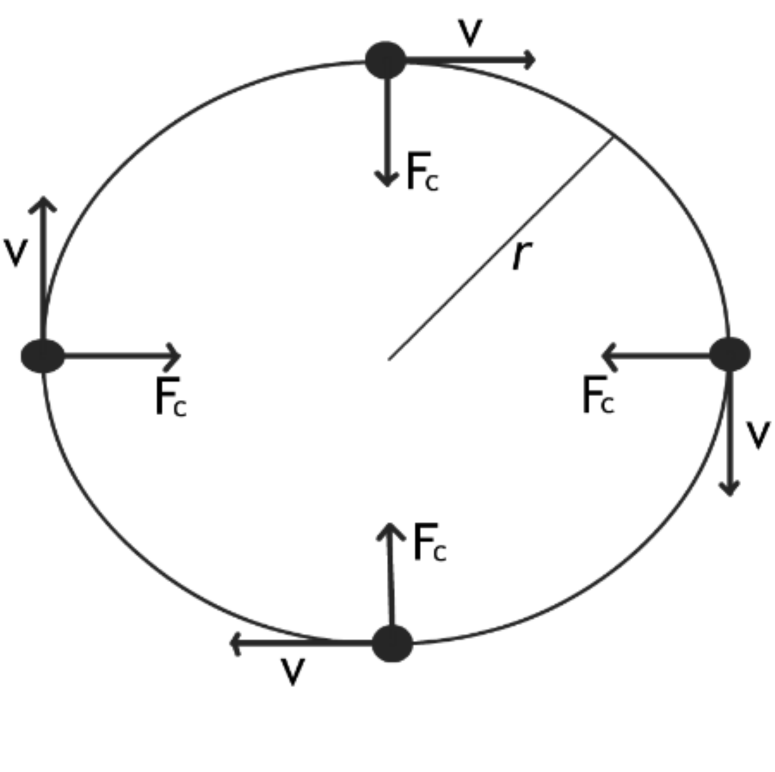
Note: If the centripetal force is no longer present, the object cannot continue to move in a circular path and instead will move along a straight line tangent to the circle.

Relating Centripetal Force with Velocity

- When an object moves along a circular path its **velocity will always be tangent to the path**, while its **centripetal force will always be perpendicular to the velocity** and **point towards the center (along the radial axis)**.



- This will always be the relationship between the centripetal force and the tangential velocity **at any point along the circular path**, as shown in the clockwise motion below:



Calculating Centripetal Force

- Hopefully we now know that when an object undergoes circular motion it must have at least one force pointing towards the center.

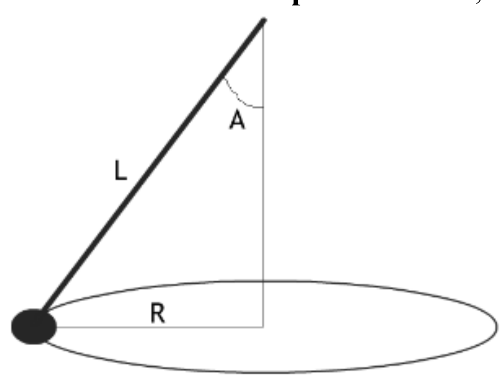
- However, that doesn't mean all the forces acting on that object will point towards the center (along the radial axis).

- Indeed, when an object moves in a circular path there may be several **forces that are perpendicular to the radial axis**, and so they **do not contribute to the centripetal force**.

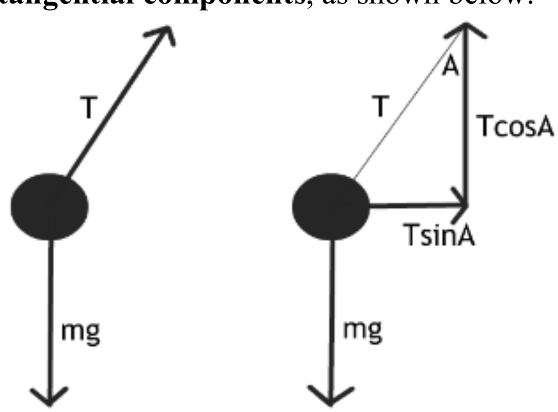
- If, however, a force points neither along the radial axis nor perpendicular to the axis, then it must be located at some angle in between the two positions.

- When this happens, you should recognize that this force vector can be broken up into **two components**: one that points along the **radial axis**, and one that points **tangent** to the circular path.

- This can more easily be understood by examining the figure below in which a **ball attached to a string** of length, ***L***, is **rotated in a circular path** of radius, ***R***:



- As you can see, the string is located at an angle, ***A***, from the vertical axis, and so the corresponding free-body diagram shows how the tension, **T**, in the string can be broken up into its **radial and tangential components**, as shown below:



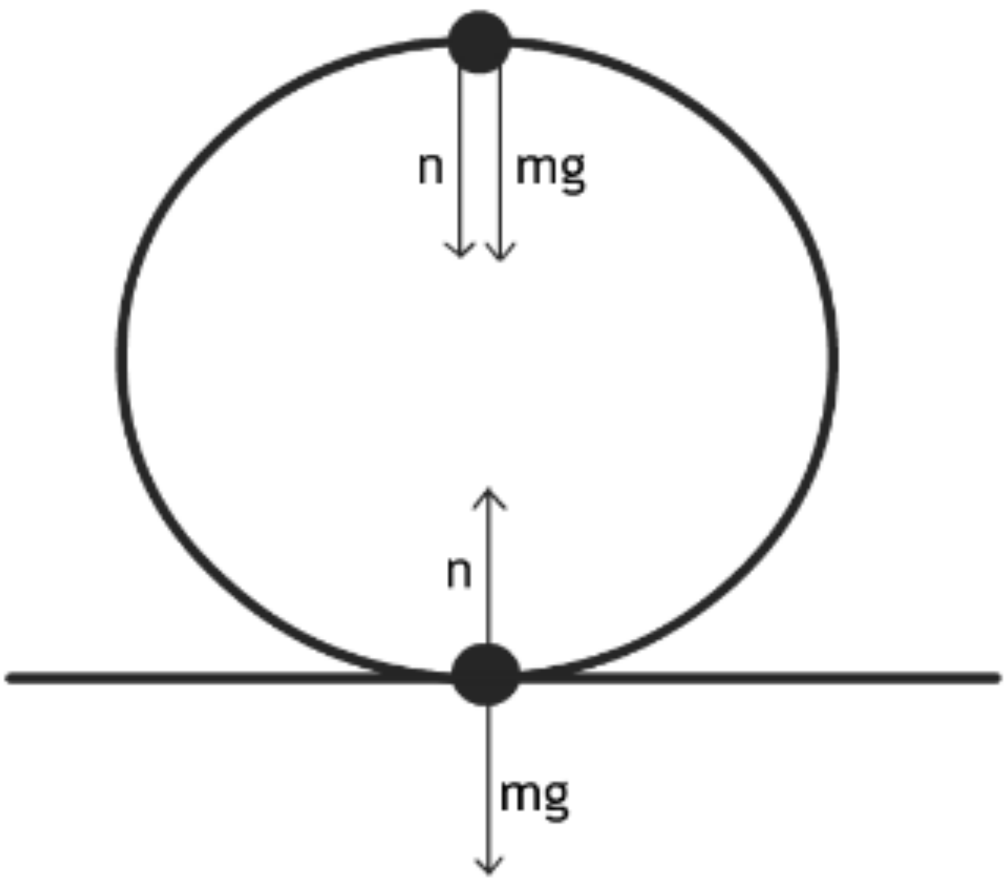
- **The vector component that points along the radial axis, TsinA, will thus be used when calculating the centripetal force**.

- It should also be noted that **any force** vector (or vector component) that points **directly opposite to the center** of the path is still located along the radial axis and **must also be considered when calculating the centripetal force**.

- Simply put, any forces (or components of forces) that point in the opposite direction of the path's center should be subtracted from the forces pointing towards the center.

- Thus, the **centripetal force** can be thought of as the **sum of all forces pointing towards the center minus the sum of all forces pointing away from the center**.

- An example of this can be seen by examining the forces shown in the image below at various points along the **vertical circular path**:

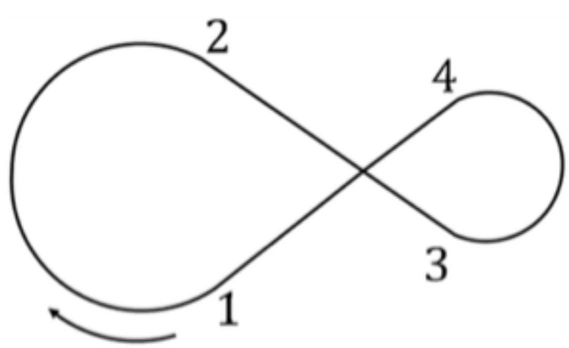


- At the **top** of the circular path the centripetal force, $\mathbf{F_{c,top} = n + mg}$
- At the **bottom** of the circular path the centripetal force, $\mathbf{F_{c,bottom} = n - mg}$

Problem 2:
A cat with mass 5 kg is tied to a rope and swung in a vertical circle with radius 50 cm. If the cat moves at a constant speed of 10 m/s, what is the tension in the rope at the top of the circular path, T_{Top}, and at the bottom of the circular path, T_{Bottom}?

A. T_{Top} = 950 N; T_{Bottom} = 950 N
B. T_{Top} = 1050 N; T_{Bottom} = 1050 N
C. T_{Top} = 950 N; T_{Bottom} = 1050 N
D. T_{Top} = 1050 N; T_{Bottom} = 950 N
E. T_{Top} = 1000 N; T_{Bottom} = 1000 N

Problem 3:
A cat moves at a constant speed around a figure-eight track in the direction shown. Where is the magnitude of the object's centripetal force the largest?



A. Between points 1 and 2
B. Between points 2 and 3, and between points 4 and 1
C. Between points 3 and 4
D. It is the same throughout
E. Depends of the magnitude of the constant speed

Solutions

ATTENTION: We have recently been informed that the data sheet on your upcoming midterm will **NOT** include the four standard **kinematic equations** for one-dimensional motion under constant acceleration (the equations on Page 4 of the Review Packet). Apparently you will be given one of the four equations and expected to derive the other three. Therefore I strongly recommend that you **memorize those four equations** and immediately write them down at the start of your exam, so that you can then use them as and when is needed, without having to derive them each and every time.

The solutions for problems 1-3 are below.

1C

The question is really asking two separate things, what is the centripetal acceleration, and what is the time required to complete one full circle?

To determine centripetal acceleration, we first recognize that it is of course not equal to zero since anytime you move in a circular path there must be a centripetal acceleration. To calculate centripetal acceleration, we use the equation $a_c = v^2/r$, and then plug in the values given for both cats.

For Cat #1 this equals $a_c = v^2/R$,

For Cat #2 this equals $a_c = (4v)^2/(4R) = 16v^2/4R = 4(v^2/R)$

Thus for Cat #2 the centripetal acceleration is four times larger than it is for Cat #1, meaning that Statement iii is correct.

We next want to determine the time it takes to complete one full circle. Since the cats both move at a constant speed, we can use the standard equation for average speed, v_{avg} = distance/time.

For both cats the distance d, can be determined by working out the circumference of the circular path, which is given by $d = 2\pi r$.

So by rearranging the equation for average speed to solve for time t, we get $t = d/v_{avg}$.

For Cat #1 this equals $t_1 = 2\pi R/v$

For Cat #2 this equals $t_2 = 2\pi(4R)/(4v) = 2\pi R/v$.

Thus both cats end up having the exact same time for completing each circle, meaning that Statement v is correct.

2C

The centripetal force for the cat moving in a vertical circular path is always equal to sum of the forces pointing towards the center minus the sum of forces pointing directing away from (or opposite to) the center. Whatever forces are used to create the centripetal force, you can always set the centripetal force equal to mv^2/r. Using this method we can set up equations for the net force at the top and the bottom and solve for the tension, T.

At the top of the circle, the forces acting on the cat, are the Tension, T_{Top}, and its weight, mg, both of which point down, towards the center. So the equation becomes:

$T_{Top} + mg = mv^2/r$,

$T_{Top} = mv^2/r - mg = 1000 - 49 = 951$ N

At the bottom of the circle, the forces acting on the cat, are the Tension, T_{Bottom}, which points upwards (towards the center), and its weight, mg, which points down (away from the center). So the equation becomes:

$T_{Bottom} - mg = mv^2/r$,

$T_{Top} = mv^2/r + mg = 1000 + 49 = 1049$ N

3C

Remember that centripetal force is equal to mv^2/r, and since the magnitude of the velocity (its speed) is constant, and the mass is also constant, the only variable that changes as the cat moves along the figure-eight track is the radius of the circular path.

Between points 2 and 3, as well as between points 4 and 1, the cat moves in a straight line, which means there is no centripetal force.

However, between points 1 and 2, and then later between points 3 and 4, the path becomes circular, with the radius of the circular path between points 1 and 2 being larger than the radius between points 3 and 4. Since radius is on the denominator when calculating centripetal force, the smaller radius will have the larger centripetal force (assuming all else remains constant), and so from points 3 to 4, where the radius is smallest, we have the largest centripetal force.