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MATH110 – Final Exam (New Material) – Sample Test Solutions

1. D

   Simplify, then use the rules of integration to take the antiderivative.

$$\int t^{3/2} - t^{1/2} dt$$
$$= \frac{t^{5/2}}{5/2} - \frac{t^{3/2}}{3/2} + C$$
$$= \frac{2}{5}t^{5/2} - \frac{2}{3}t^{3/2} + C$$
$$= 2t^{3/2}\left(\frac{1}{5}t - \frac{1}{3}\right) + C$$

2. B.

   Take the integral, then use the fixed costs to solve for C. If fixed costs are $400, then that means C(0)=$400.

$$C(x) = \int \left(-0.4x + 100\right) dx$$
$$C(x) = -0.4\frac{x^2}{2} + 100x + C$$
$$C(x) = -0.2x^2 + 100x + C$$
$$400 = 0 + 0 + C$$
$$C = 400$$
$$C(x) = -0.2x^2 + 100x + 400$$
$$C(100) = -0.2(100)^2 + 100(100) + 400$$
$$C(100) = -0.2(10{,}000) + 10{,}000 + 400$$
$$C(100) = -2000 + 10{,}000 + 400$$
$$C(100) = 8400$$

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3. B

Use u-substitution to take the integral. Let $u = x^3$.

$$\int 3x^2 e^u \frac{du}{3x^2}$$

$$\int e^u \, du$$

$$e^u$$

$$e^{x^3}\Big|_0^1 = e^1 - e^0 = e - 1$$

$$u = x^3$$

$$du = 3x^2 dx$$

$$dx = \frac{du}{3x^2}$$

4. E

Use the average value formula.

$$\frac{1}{3-0}\int_0^3 te^{t^2} \, dt$$

$$\frac{1}{3}\int_0^3 te^u \frac{du}{2t}$$

$$\frac{1}{3}\int_0^3 \frac{1}{2}e^u \, du$$

$$\frac{1}{6}\int_0^3 e^u \, du$$

$$\frac{1}{6}e^u = \frac{1}{6}e^{t^2}\Big|_0^3$$

$$\frac{1}{6}(e^{3^2} - e^{0^2}) = \frac{1}{6}(e^9 - 1)$$

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5. A.
This is a producer surplus question. Start by setting the equation equal to each other to obtain x and p.

$$70 - 0.1x = 20 + 0.4x$$

$$50 = 0.5$$

$$x = 100$$

$$p = 70 - .1(100) = 60$$

$$PS = (100)(60) - \int_0^{100} \left(S(x)\right)dx$$

$$= 6000 - \int_0^{100} \left(20 + .4x\right)dx$$

$$= 6000 - \left[20x + .2x^2\right]\Big|_0^{100}$$

$$= 6000 - [(20)(100) + .2(100)^2 - 0]$$

$$= 6000 - [2000 + 2000]$$

$$= 2000$$

6. C
This is an area between two curves problem

$$x^4 = 1$$

$$x = 1, -1$$

$$\int_{-1}^{1} (1 - x^4)dx = x - \frac{x^5}{5}\Big|_{-1}^{1}$$

$$= \left[1 - \frac{1}{5}\right] - \left[-1 - \frac{-1}{5}\right] = 1 - \frac{1}{5} + 1 - \frac{1}{5} = 2 - \frac{2}{5} = 8/5$$

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7. D

Use the FV of an Income Stream formula.

$$e^{.10(20)}\int_0^{20} 8000e^{-.1t}\,dt$$

$$= e^{2}\int_0^{20} 8000e^{-.1t}\,dt$$

8. A

Use the formula for the amount of an annuity.

$$\frac{2400}{.07}(e^{0.7}-1)$$

9. B

Start by taking the integral, using a u-substitution, then use the information given to solve for C.

$$p(x)=\int \frac{-500x}{(9+x^2)^{3/2}}\,dx$$

$$u = 9+x^2$$

$$du = 2xdx$$

$$dx = \frac{du}{2x}$$

$$p(x)=\int \frac{-500x}{(u)^{3/2}}\frac{du}{2x}$$

$$p(x)=\int -250u^{-3/2}\,du$$

$$p(x)=-250\frac{u^{-1/2}}{-1/2}+C$$

$$p(x)=\frac{500}{\sqrt{u}}+C$$

$$p(x)=\frac{500}{\sqrt{9+x^2}}+C$$

$$130=\frac{500}{\sqrt{9+(4)^2}}+C$$

$$130=100+C$$

$$C=30$$

$$p(x)=\frac{500}{\sqrt{9+x^2}}+30$$

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10. E

Take the integral to find the Revenue function. Then use the point $R(0) = 0$ to solve for C. Then plug in 100 for x.

$$R(x) = 10\frac{e^{2x}}{2} + 2\frac{x^2}{2} + C$$

$$R(x) = 5e^{2x} + x^2 + C$$

$$0 = 5(e)^0 + (0) + C$$

$$0 = 5 + C$$

$$C = -5$$

$$R(x) = 5e^{2x} + x^2 - 5$$

11. B

We are given that:

$$P'(x) = -2x + 20$$

$$P(0) = -40$$

We want to know the maximum profit. Remember that profit is maximized so we will set $P'(x) = 0$ to find out what number we ultimately need to plug into our profit equation.

$$-2x + 20 = 0$$

$$x = 10$$

$$P(x) = \int -2x + 20dx$$

$$P(x) = -x^2 + 20x + C$$

Use $P(0) = -40$ to solve for C.

$$-40 = -(0)^2 + 20(0) + C$$

$$C = -40$$

$$P(x) = -x^2 + 20x - 40$$

$$P(10) = -100 + 200 - 40 = 60$$

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12. E

First find the intersection points.

$x^2 = \sqrt{x}$

$x^4 = x$

$x = 0,1$

Next, you know that $\sqrt{x}$ is the top curve. You can figure this out visually by graphing, or you can plug in a number between 0 and 1 into both functions to see which function gives you the bigger value.

Now we integrate…

$$\int_0^1 x^{1/2} - x^2 dx$$

$$= \frac{2}{3}x^{3/2} - \frac{1}{3}x^3 \Big|_0^1$$

$$= \left(\frac{2}{3} - \frac{1}{3}\right) - (0-0) = \frac{1}{3}$$

13. A

Use the average value formula. You will have to use substitution to integrate.

$$\frac{1}{3-1}\int_1^3 \frac{2t}{t^2+1} dt$$

$$\frac{1}{2}\int_1^3 \frac{2t}{u}\frac{du}{2t}$$

$$\frac{1}{2}\int_1^3 u^{-1}\, du$$

$$\frac{1}{2}\ln u = \frac{1}{2}\ln(t^2+1)\Big|_1^3$$

$$\frac{1}{2}\ln(3^2+1) - \frac{1}{2}\ln(1^2+1)$$

$$\frac{1}{2}\ln 10 - \frac{1}{2}\ln 2 = \frac{1}{2}\ln 5$$

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14. B

Use substitution.

$u = 1 + e^{5x}$

$du = 5e^{5x}dx$

$dx = \frac{du}{5e^{5x}}$

$\int \frac{e^{5x}}{u} \frac{du}{5e^{5x}}$

$\int \frac{1}{5} \bullet \frac{1}{u} du$

$\frac{1}{5} \ln u$

$\frac{1}{5} \ln(e^{5x} + 1) + C$

15. C

Use substitution.

$u = \ln x$

$du = \frac{1}{x} dx$

$dx = xdu$

$\int \frac{u^2}{x} x\, du$

$\int u^2\, du$

$\frac{u^3}{3} = \left. \frac{(\ln x)^3}{3} \right|_1^e$

$\frac{(\ln e)^3}{3} - \left(\frac{(\ln 1)^3}{3} \right) = \frac{1}{3} - 0$

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16. D

Use u-substitution to take the integral, then use the given point to solve for C.

$u = \ln x$

$du = \frac{1}{x} dx$

$dx = xdu$

$f(x) = \int \frac{u}{x} x \, du$

$f(x) = \int u \, du$

$f(x) = \frac{u^2}{2} + C$

$f(x) = \frac{1}{2} u^2 + C$

$f(x) = \frac{1}{2} (\ln x)^2 + C$

$2 = \frac{1}{2} (\ln 1)^2 + C$

$2 = 0 + C$

$C = 2$

$f(x) = \frac{1}{2} (\ln x)^2 + 2$

17. B

You will need to use substitution, then go back and solve for x in terms of u.

$u = x - 2$

$du = dx$

$x = u + 2$

$\int \frac{x}{u} du$

$\int \frac{u+2}{u} du$

$\int \frac{u}{u} + \frac{2}{u} du$

$\int 1 + 2u^{-1} \, du$

$u + 2 \ln u + C$

$x - 2 + 2 \ln(x - 2) + C$

$x + 2 \ln(x - 2) + C$

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18. C

Use u-substitution to integrate. Then evaluate from 0 to 2.

$u = 1 + 2x^2$

$du = 4xdx$

$dx = \frac{du}{4x}$

$\int \frac{8x}{\sqrt{u}} \frac{du}{4x}$

$\int 2u^{-1/2}\, du$

$2\frac{u^{1/2}}{1/2} = 4\sqrt{u} = 4\sqrt{1+2x^2}\Big|_0^2$

$= \left(4\sqrt{1+2(2)^2}\right) - \left(4\sqrt{1+2(0)^2}\right)$

$= 4\sqrt{9} - 4\sqrt{1} = 12 - 4 = 8$

19. D

Use the formula for Value of an Income Stream.

$e^{.05(8)} \int_0^8 2000e^{-.05t}\, dt$

$= e^{.4} \int_0^8 2000e^{-.05t}\, dt$

20. A

Use the formula for PV of an Income Stream

$\int_0^T R(t)e^{-rt}\, dt$

$\int_0^{12} 6000e^{-.05t}\, dt$

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21. B

Start by finding the equilibrium quantity and price and then use the formula for Consumer Surplus.

$$20-\frac{1}{3}x=12+x$$

$$8=\frac{4}{3}x$$

$$x=6$$

$$p=12+6=18$$

$$CS=\int_0^6\left(20-\frac{1}{3}x\right)dx-108$$

$$CS=\left[20x-\frac{1}{6}x^2\right]_0^6-108$$

$$CS=\left[\left(120-6\right)-(0)\right]-108$$

$$CS=114-108=6$$

22. C

Use the formula for PV of an annuity.

23. B

When a question asks "find the area of the region under the graph" that just means to integrate. There are two methods for integrating this function. You can use integration by parts with $u=\ln x$, $dv=dx$. Or you can just memorize that $\int \ln x\,dx = x\ln x - x$

$$\int_1^4 \ln x\,dx = x\ln x - x\Big|_1^4$$

$$=\left[4\ln(4)-4\right]-\left[1\ln(1)-1\right]$$

$$=4\ln(4)-4-0+1$$

$$=4\ln(4)-3$$

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24. D

Integrate using substitution.

$$\int \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$$

$$\int \frac{e^{u}}{\sqrt{x}}(-2\sqrt{x}\,du)$$

$$\int -2e^{u}\,du$$

$$-2e^{u}$$

$$-2e^{-\sqrt{x}}\Big|_{1}^{\infty}$$

$$-2e^{-\sqrt{\infty}}+2e^{-\sqrt{1}}=-2(0)+2e^{-1}=2/e$$

25. C

$$\int_{9}^{\infty}\frac{6}{x^{3/2}}\,dx=\int 6x^{-3/2}\,dx$$

$$=\frac{6x^{-1/2}}{-1/2}$$

$$=\frac{-12}{\sqrt{x}}\Big|_{9}^{\infty}$$

$$=\frac{-12}{\sqrt{\infty}}+\frac{12}{\sqrt{9}}$$

$$=0+12/3=4$$

26. E

$$\int_{1}^{\infty}2x^{-1/2}\,dx=$$

$$=\frac{2x^{1/2}}{1/2}$$

$$=4\sqrt{x}\Big|_{1}^{\infty}$$

$$=4\sqrt{\infty}-4\sqrt{1}$$

$$=\infty-1$$

$$=\infty$$

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27. B

Since normal u-substitution does not work, you have to use integration by parts.

$$\int xe^{2x}\,dx = uv - \int v\,du$$

$$2x(e^x) - \int e^x 2\,dx \qquad u = 2x \qquad v = e^x$$

$$2xe^x - 2\int e^x\,dx \qquad du = 2dx \qquad dv = e^x dx$$

$$2xe^x - 2e^x + C$$

28. D

Since normal u-substitution does not work, you have to use integration by parts.

$$\int x\ln x = uv - \int v\,du$$

$$\ln x\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2}\frac{1}{x}dx \qquad u = \ln x \qquad v = \frac{x^2}{2}$$

$$\frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx \qquad du = \frac{1}{x}dx \qquad dv = xdx$$

$$\frac{1}{2}x^2\ln x - \frac{1}{2}\left(\frac{x^2}{2}\right) + C$$

29. A

Use the formula for a perpetuity.

$$PV = \frac{mP}{r}$$

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