MATH022 Final Exam (New Material) – Sample Test – Detailed Solutions

1. B.

$y = b^x$

$5 = b^{-1/2}$

$5 = \frac{1}{b^{1/2}}$

$5 = \frac{1}{\sqrt{b}}$

$5\sqrt{b} = 1$

$\sqrt{b} = \frac{1}{5}$

$b = \frac{1}{25}$

2. C

$\left(2^3\right)^x = \left(2^{-1}\right)^{x+5}$

$2^{3x} = 2^{-(x+5)}$

$3x = -x - 5$

$4x = -5$

$x = -5/4$

3. E

Answer choice E is false.

To find the y-intercept, you set $x = 0$.

This gives us $y = \left(\frac{1}{3}\right)^0 + 1 = 1 + 1 = 2$

So the y-intercept would be $(0,2)$

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4. C

$$\frac{1}{e^x} = \frac{e^{1/2}}{e^{1-x}}$$

$$e^{-x} = e^{(1/2-(1-x))}$$

$$-x = \frac{1}{2} - 1 + x$$

$$\left(-x = \frac{1}{2} - 1 + x\right)2$$

$$-2x = 1 - 2 + 2x$$

$$-2x = -1 + 2x$$

$$4x = 1$$

$$x = 1/4$$

5. B

Answer choice B is false. The range will be $(-\infty, 2)$ since the graph gets flipped over the x-axis.

6. D

Simplify using rules of logs and exponentials.

$$\log \sqrt[e]{10}$$

$$= \log_{10} \sqrt[e]{10}$$

$$= \log_{10} (10)^{1/e}$$

$$= \frac{1}{e} \log_{10} 10$$

$$= \frac{1}{e}(1) = \frac{1}{e}$$

7. E

Since all the answer choices are in terms of ln's it makes sense to use the change of base formula here in order to get a common base.

$$\log_{1/3} 5$$

$$= \frac{\ln 5}{\ln(1/3)}$$

$$= \frac{\ln 5}{\ln 1 - \ln 3}$$

$$= \frac{\ln 5}{0 - \ln 3} = -\frac{\ln 5}{\ln 3}$$

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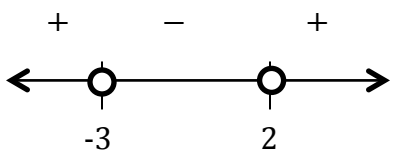
8. A

The rule for domain of logarithms is that whatever is inside the log must be > 0. To solve this inequality set the top and bottom equal to zero, then set up a number line.

$$\frac{x-2}{x+3} > 0$$



9. D

Convert to an exponential, then solve for x by equating the bases.

$$\log_{\sqrt{2}} \frac{1}{8} = x$$

$$\left(\sqrt{2}\right)^x = \frac{1}{8}$$

$$\left(2^{1/2}\right)^x = \frac{1}{2^3}$$

$$2^{x/2} = 2^{-3}$$

$$\frac{x}{2} = -3$$

$$x = -6$$

10. B

Convert to an exponential since we cannot equate the bases…

$$\log_{\sqrt{2}} 3 = x$$

$$\left(\sqrt{2}\right)^x = 3$$

$$\left(2^{1/2}\right)^x = 3$$

$$2^{x/2} = 3$$

$$\log_2 3 = \frac{x}{2}$$

$$x = 2\log_2 3$$

$$x = \log_2\left(3^2\right)$$

$$x = \log_2 9$$

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11. C

Take the ln of both sides, use rules of logs to bring down the exponents, distribute the $1-2x$, get all the x's on one side, then solve for x.

$$3^{1-2x} = 7^x$$

$$\ln 3^{1-2x} = \ln 7^x$$

$$(1-2x)\ln 3 = x\ln 7$$

$$\ln 3 - 2x\ln 3 = x\ln 7$$

$$\ln 3 = x\ln 7 + 2x\ln 3$$

$$\ln 3 = x(\ln 7 + 2\ln 3)$$

$$x = \frac{\ln 3}{2\ln 3 + \ln 7}$$

12. E

The answer is "No Solution" because an exponential will never equal a negative number.

13. A

Use rules of exponentials to combine to a single e on the left side. Then take the ln of both sides to solve.

$$e^{(3x+5)-(2x-1)} = 8$$

$$e^{3x+5-2x+1} = 8$$

$$e^{x+6} = 8$$

$$\ln e^{x+6} = \ln 8$$

$$x+6 = \ln 8$$

$$x = \ln 8 - 6$$

14. D

Use rules of logs to combine to a single log on the left side. Then covert to an exponential and solve for x. Be sure to check the domain!

$$\log_3(x(x-2)) = 1$$

$$3^1 = x(x-2)$$

$$3 = x^2 - 2x$$

$$0 = x^2 - 2x - 3$$

$$0 = (x-3)(x+1)$$

$$x = 3, x = -1$$

$x = -1$ is not in the domain, so the only answer is $x = 3$.

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15. B

Get all the logs on one side and combine to a single log. Then convert to an exponential and solve.

$$\log_2(x-5)+\log_2(x+6)=\log_2 x+1$$

$$\log_2(x-5)+\log_2(x+6)-\log_2 x=1$$

$$\log_2\left(\frac{(x-5)(x+6)}{x}\right)=1$$

$$2^1=\frac{(x-5)(x+6)}{x}$$

$$2x=x^2+x-30$$

$$0=x^2-x-30$$

$$0=(x-6)(x+5)$$

$$x=6, x=-5$$

$x=-5$ is not in the domain, so the only answer is $x=6$.

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