MATH110 – Exam 2 - Sample Test 1 – Detailed Solutions

1. B

   Create a first derivate number line.

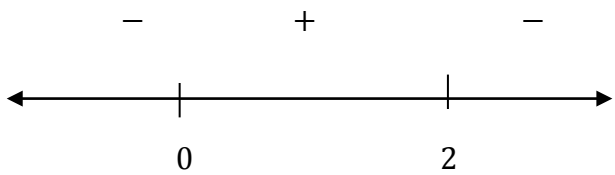
   $f'(x) = x^6(-3e^{-3x}) + (e^{-3x})6x^5$

   $f'(x) = -3x^6e^{-3x} + 6x^5e^{-3x}$

   $0 = 3x^5e^{-3x}(-x+2)$

   $0 = 3x^5e^{-3x}(2-x)$

   $x = 0, x = 2$



   Remember that e raised to any power will always be positive.

2. A

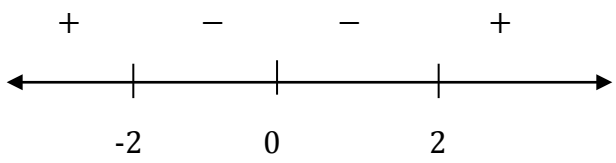
   Create a first derivative number line.

   $f'(x) = 15x^4 - 60x^2$

   $0 = 15x^2(x^2-4)$

   $0 = 15x^2(x-2)(x+2)$

   $x = 0, x = 2, x = -2$



3. E

   Take the derivate using the quotient rule and then set the top and bottom equal to 0. Remember that critical numbers must be in the domain of $f(x)$, so we can immediately cross off any answer choices that contain $x = 2$.

   $f'(x) = \dfrac{(x-2)5e^{4x}(4) - 5e^{4x}(1)}{(x-2)^2}$ Quotient Rule

   $f'(x) = \dfrac{5e^{4x}\left(4(x-2)-1\right)}{(x-2)^2}$ Factor out $5e^x$

   $f'(x) = \dfrac{5e^{4x}\left(4x-9\right)}{(x-2)^2}$ Simplify

   $5\cancel{e^{4x}}\left(4x-9\right) = 0$ $\quad$ $\left(x-2\right)^2 = 0$

   $4x - 9 = 0$ $\quad$ $x - 2 = 0$ $\quad$ Set top and bottom equal to 0.

   $x = 9/4$ $\quad$ $\cancel{x = 2}$

   Remember that e^x will never equal 0, and $x = 2$ is not in the domain. So $x = 9/4$ is the only critical number.

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4. B

To find the H.A, look at the powers on top and bottom. Since the powers are the same, divide the coefficients.

So the H.A. is $y = -4$.

To find the V.A., simplify the function, and then set the denominator equal to 0.

$$f(x) = \frac{4-4x^2}{x^2+4x+3} = \frac{4(1-x)(1+x)}{(x+3)(x+1)}$$

$$f(x) = \frac{4(1-x)}{(x+3)}$$

V.A. $x = -3$

5. A

Create the second derivative number line by setting the top and the bottom of the second derivative equal to 0.

$x^2 - 16 = 0$

$x^2 = 16$

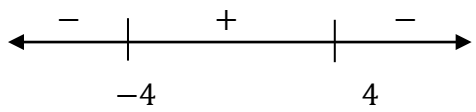
$x = 4, x = -4$

$-3(x^2+25) = 0$

$x^2 + 25 = 0$

$x^2 = -25$

no solution



6. D

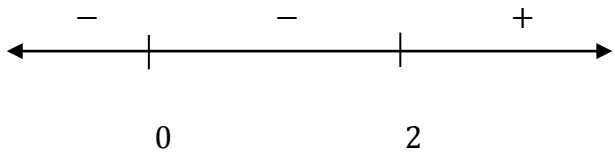
Create a second derivative number line. Inflection points occur anywhere the concavity changes.

$f'(x) = 15x^4 - 40x^3 + 8$

$f''(x) = 60x^3 - 120x^2$

$0 = 60x^2(x-2)$

$x = 0, x = 2$



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7. E

Take the first derivative, using the quotient, to find the critical numbers. Remember that you only want to look at critical numbers that are within the interval of [0,3]. Then take the critical numbers, and the endpoints of the interval and plug them into the original function. The highest value will be your maximum, and the lowest value will be your minimum.

$$f'(x) = \frac{(x^2+1)5 - 5x(2x)}{(x^2+1)^2}$$

$$f'(x) = \frac{5x^2+5-10x^2}{(x^2+1)^2}$$

$$f'(x) = \frac{5-5x^2}{(x^2+1)^2}$$

$5-5x^2=0$	$(x^2+1)^2=0$
$5=5x^2$	$x^2+1=0$
$x^2=1$	$x^2=-1$
$x=1, \cancel{x=-1}$	*No Solution*

$$f(0)=0$$
$$f(1)=5/2$$
$$f(3)=3/2$$

8. B

To find the equation of the tangent line you need a point and a slope. The point is given. Find the slope by taking the derivative using the product rule, then plugging in 1.

Point: (1,0)
Slope: e

$$f'(x) = e^x\left(\frac{1}{x}\right) + \ln x\left(e^x\right)$$

$$f'(1) = e^1(1) + \ln(1)\left(e^1\right)$$

$$f'(1) = e$$

Equation:

$$y - 0 = e(x-1)$$

$$y = ex - e$$

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9. A

To find the point of diminishing returns take the second derivative and set it equal to 0. Remember that the answer will be in thousands.

$P'(x) = -3x^2 + 12x + 100$

$P''(x) = -6x + 12$

$0 = -6x + 12$

$6x = 12$

$x = 2$

Note that $x = 2$ is a point of diminishing returns because the function goes from concave up to concave down at this point.

10. C

Set up an equation for both scenarios and set them equal to one another. Remember it doesn't matter what t equals since it is the same on both sides, so you can just cross the t's out.

$10{,}000\left(1 + \frac{.08}{2}\right)^2 = 10{,}000e^r$

$(1.04)^2 = e^r$

$\ln(1.04)^2 = r$

$r = 2\ln(1.04)$

11. C

This is a business optimization question. Start by finding the profit equation, then take the derivative and set it equal to 0.

$P(x) = px - C$

$P(x) = (250 - x)x - (10x + 500)$

$P(x) = 250x - x^2 - 10x - 500$

$P(x) = 240x - x^2 - 500$

$P'(x) = 240 - 2x$

$0 = 240 - 2x$

$2x = 240$

$x = 120$

$p(x) = 250 - 120 = \$130$

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12. A

Use the formula for compound interest:

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$A = 2000\left(1+\frac{.10}{4}\right)^{(4)(5)}$$

$$A = 2000(1.025)^{20}$$

13. E

Use $A = Pe^{rt}$ and solve for r.

$$7000 = 5000e^{3r}$$
$$\frac{7}{5} = e^{3r}$$
$$\ln\left(\frac{7}{5}\right) = 3r$$
$$r = \frac{\ln\left(\frac{7}{5}\right)}{3}$$

14. D

Simplify first using rules of logs, then take the derivative.

$$f(x) = \ln\sqrt{x^2+6x+100} \ = \ln(x^2+6x+100)^{\left(\frac{1}{2}\right)} = \frac{1}{2}\ln(x^2+6x+100)$$
$$f'(x) = \frac{1}{2}\times\frac{1}{x^2+6x+100}\times(2x+6) = \frac{2(x+3)}{2(x^2+6x+100)} = \frac{(x+3)}{(x^2+6x+100)}$$

15. B

Start by finding the revenue equation, then simplify and take the derivative using the product rule.

$$R(x) = px$$
$$R(x) = (x^2e^{3x}+1)x$$
$$R(x) = x^3e^{3x}+x$$
$$R'(x) = x^3e^{3x}3+e^x3x^2+1$$ Take the derivative. Use product rule on the first term.

$$R'(x) = 3x^2e^x(x+1)+1$$ Factor x^2e^x out of the first two terms.

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16. B

Use the Present Value formula continuous compounding.

$P = Ae^{-rt}$

$P = 10{,}000e^{-.08(4)}$

$P = 10{,}000e^{-0.32}$

17. E

Answer choice E is false because $\lim_{x \to -\infty} 2e^x = 0$.

All other answer choices are true. They are statements that come directly from the lectures notes and need to be memorized.

18. B

To find $s(0)$, plug in 0 for t. Remember that $e^0 = 1$.

To find the carrying capacity, factor out the bottom and simplify. Then carrying capacity is equal to the number on top.:

$$s(t) = \frac{9000}{3+6e^{-3t}} = \frac{9000}{3(1+2e^{-3t})} = \frac{3000}{1+2e^{-3t}}$$

19. D

Use logarithmic differentiation, then plug in 1.

$y = x^{x^2+3}$

$\ln y = \ln x^{(x^2+3)}$

$\ln y = (x^2+3)\ln x$

$\frac{1}{y}y' = (x^2+3)\frac{1}{x} + \ln x(2x)$

$y' = \left((x^2+3)\frac{1}{x} + \ln x(2x)\right)y$

$y' = \left((x^2+3)\frac{1}{x} + \ln x(2x)\right)x^{(x^2+3)}$

$y'(1) = \left(4 + 2\ln(1)\right)(1)^{(4)} = 4$

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20. A

This is an optimization question. You want to minimize cost. Use the volume to solve for y.

$$x^2 y = 8$$

$$y = \frac{8}{x^2}$$

$$C = 1x^2 + 5x^2 + 4(3)xy$$

$$C = 6x^2 + 12x\left(\frac{8}{x^2}\right)$$

$$C = 6x^2 + 96x^{-1}$$

$$C' = 12x - 96x^{-2}$$

$$0 = 12x - \frac{96}{x^2}$$

$$12x = \frac{96}{x^2}$$

$$12x^3 = 96$$

$$x^3 = 8$$

$$x = 2$$

$$y = \frac{8}{x^2}$$

$$y = 2$$

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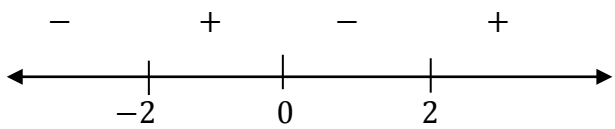
1. E
   Create a first derivative number line.

   $f'(x) = 4x^3 - 16x$

   $0 = 4x(x^2 - 4)$

   $0 = 4x(x-2)(x+2)$



2. A
   Create a first derivative number line. Remember to use the product rule on the first term when you take the derivative.

   $f'(x) = 10x^2e^x + e^x(20x) + 0$

   $f'(x) = 10x^2e^x + 20xe^x$ Simplify

   $f'(x) = 10xe^x(x+2)$ Factor out $10xe^x$

   $0 = 10xe^x(x+2)$ Set the derivative equal to 0.

   $10x = 0, \quad e^x = 0, \quad x+2 = 0$

   $x = 0, \ \textit{No Solution}, x = -2$

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3. B

Take the derivate, find a common denominator, then set the top and bottom equal to 0.

$$f'(x) = 2x - \frac{50}{x}$$

$$f'(x) = \frac{2x^2 - 50}{x}$$

$$2x^2 - 50 = 0, x = 0$$

$$x = 5, -5, 0$$

However, -5 and 0 cannot be critical numbers because they are not in the domain of the original function.

4. B

To find the H.A., look at the powers. Since the powers are the same you can divide the coefficients. This gives us a H.A. of $y = 2$.

To find the V.A., start by simplifying the function. Then, set the denominator equal to.

$$f(x) = \frac{2x^2 + 14x + 24}{x(x+3)} = \frac{2(x^2 + 7x + 12)}{x(x+3)} = \frac{2\cancel{(x+3)}(x+4)}{x\cancel{(x+3)}}$$

This gives us a V.A. of $x = 0$.

5. A

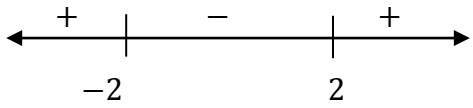
Create the second derivative number line by setting the top and the bottom of the second derivative equal to 0.

$$x^2 - 4 = 0 \qquad 5(x^2 + 0) = 0$$

$$x^2 = 4 \qquad x^2 + 9 = 0$$

$$x = 2, x = -2 \qquad x^2 = -9$$

no solution



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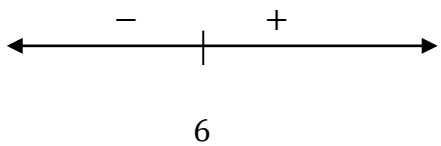
6. C

Start by taking the second derivative and setting it equal to 0. Remember that you can never take the log of a negative number or 0!

$$f'(x) = 72\frac{1}{x} + 2x$$
$$f''(x) = -\frac{72}{x^2} + 2$$
$$\frac{72}{x^2} = 2$$
$$2x^2 = 72$$
$$x^2 = 36$$
$$x = 6, -6$$

Since you cannot take the log of a negative number, $x = 6$ is the only potential inflection point. We just need to make sure the concavity changes there by putting it on a second derivative number line.



7. B

Take the first derivative, using the quotient, to find the critical numbers that are within the given interval. Then take the critical numbers, and the endpoints of the interval and plug them into the original function. The highest value will be your maximum, and the lowest value will be your minimum.

$f'(x) = 3x^2 + 6x$

$f'(x) = 3x(x+2)$

$2x(x+2) = 0$

$x = 0, x = -2$

$f(-3) = -1$

$f(-2) = 3$

$f(0) = -1$

$f(1) = 3$

$f(2) = 19$

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8. D

Take the derivative and plug in 3/2 to find the slope. Then plug into point slope form

$$y' = e^{2x-3}(2)$$
$$y'(3/2) = e^{2(3/2)-3}(2)$$
$$y'(3/2) = e^{0}2$$
$$y'(3/2) = 2$$

$$y - 1 = 2(x - 3/2)$$
$$y - 1 = 2x - 3$$
$$y = 2x - 2$$

9. A

To find the point of diminishing returns take the second derivative and set it equal to 0. Remember that the answer will be in thousands.

$$P'(x) = -15x^2 + 60x + 500]$$
$$P''(x) = -30x + 60$$
$$0 = -30x + 60$$
$$30x = 60$$
$$x = 2$$

Note that $x = 2$ is a point of diminishing returns because the function goes from concave up to concave down at this point.

10. B

Set up an equation for both scenarios and set them equal to one another. Remember it doesn't matter what t equals since it is the same on both sides, so you can just cross the t's out.

$$6000\left(1 + \frac{.08}{4}\right)^4 = 6000e^r$$

$$(1.02)^4 = e^r$$
$$\ln(1.02)^4 = r$$
$$r = 4\ln(1.02)$$

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11. E

This is a business optimization question. Start by finding the profit equation, then take the derivative and set it equal to 0.

$P(x) = x(500 - x) - (20x + 300)$

$P(x) = 500x - x^2 - 20x - 300$

$P(x) = -x^2 + 480x - 300$

$P'(x) = -2x + 480$

$0 = -2x + 480$

$x = 240$

$p(240) = 500 - 240 = 260$

12. C

This statement is not true. You cannot expand when you are adding or subtracting inside of a log.

13. A

Since this is a continuous interest problem, you can use the shortcut below.
Use $A = Pe^{rt}$ and solve for r.

$7000 = 4000e^{5r}$

$\frac{7}{4} = e^{5r}$

$\ln\left(\frac{7}{4}\right) = 5r$

$r = \frac{\ln\left(\frac{7}{4}\right)}{5}$

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14. ANSWER NOT LISTED

Take the derivative using the chain rule

$$f(x) = (2 + e^{-3x})^{-5}$$

$$f'(x) = -5(2 + e^{-3x})^{-6}(-3e^{-3x})$$

$$f'(x) = \frac{15e^{-3x}}{(2 + e^{-3x})^{6}}$$

15. B

Find the revenue equation, then take the derivative.

$$R(x) = \frac{\ln(3x^2 + 5)}{x} * x$$

$$R(x) = \ln(3x^2 + 5)$$

$$R'(x) = \frac{1}{3x^2 + 5}(6x)$$

16. B

Use the Present Value formula continuous compounding.

$$P = Ae^{-rt}$$

$$P = 70{,}000e^{-.05(3)}$$

$$P = 70{,}000e^{-0.15}$$

17. D

Simplify first using rules of logs, then take the derivative.

$$f(x) = \frac{1}{2}\ln(e^x + x)$$

$$f'(x) = \frac{1}{2}\frac{1}{e^x + x}(e^x + 1)$$

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18. C

To find $s(0)$, plug in 0 for t. Remember that $e^0 = 1$.

To find the carrying capacity just memorize that the carrying capacity is represented by the numerator of a logistical growth function.

19. E

Use logarithmic differentiation. Start by taking the ln of both sides, and simplifying using rules of logs.

$$\ln y = \ln\left[\frac{\left(x^2-3\right)^5}{\left(x^3+7\right)^4}\right]$$

$$\ln y = 5\ln(x^2-3) - 4\ln(x^3+7)$$

$$\frac{1}{y}y' = 5\frac{1}{x^2-3}(2x) - 4\frac{1}{x^3+7}(3x^2)$$

$$y' = \left[\frac{10x}{x^2-3} - \frac{12x^2}{x^3+7}\right]y$$

$$y' = \left[\frac{10x}{x^2-3} - \frac{12x^2}{x^3+7}\right]\frac{\left(x^2-3\right)^5}{\left(x^3+7\right)^4}$$

20. B

This is an optimization question.

$$A = (x-1)(y-6)$$
$$A = (x-1)\left(\frac{600}{x} - 6\right)$$
$$A = 600 - 6x - 600x^{-1} + 6$$
$$A' = -6 + \frac{600}{x^2}$$
$$0 = -6 + \frac{600}{x^2}$$
$$6x^2 = 600$$
$$x = 10$$

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