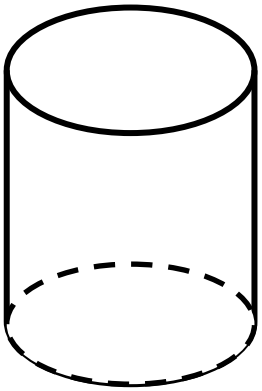
LionTutors

www.LionTutors.com

Problem 43

$Surface\ Area = \pi r^2 + 2\pi rh$

$Volume = \pi r^2 h$



$V = \pi r^2 h$

$V = \pi r^2 \left(\frac{24\pi - \pi r^2}{2\pi r} \right)$

$V = r\left(\frac{24\pi - \pi r^2}{2} \right)$

$V = 12\pi r - \frac{1}{2}\pi r^3$

$V' = 12\pi - \frac{3}{2}\pi r^2$

$0 = 12\pi - \frac{3}{2}\pi r^2$

$12\pi = \frac{3}{2}\pi r^2$

$12 = \frac{3}{2} r^2$

$r^2 = 8$

$r = \sqrt{8}$

$24\pi = \pi r^2 + 2\pi rh$

$2\pi rh = 24\pi - \pi r^2$

$h = \frac{24\pi - \pi r^2}{2\pi r}$

$h = \frac{24\pi - \pi(\sqrt{8})^2}{2\pi r} = \frac{24\pi - 8\pi}{2\pi\sqrt{8}}$

$h = \frac{16\pi}{2\pi\sqrt{8}} = \frac{8}{\sqrt{8}}$

$h = \sqrt{8}$

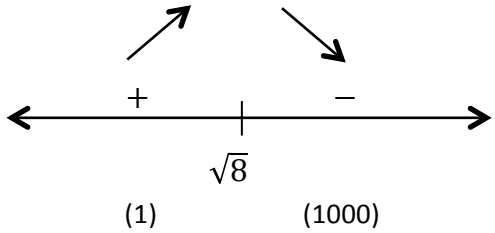
© LionTutors 2017

$$V = \pi r^2 h$$

$$V = \pi\left(\sqrt{8}\right)^2 \sqrt{8} = 8\sqrt{8}\pi = 16\sqrt{2}\pi \ \ ft^3$$

Verification:

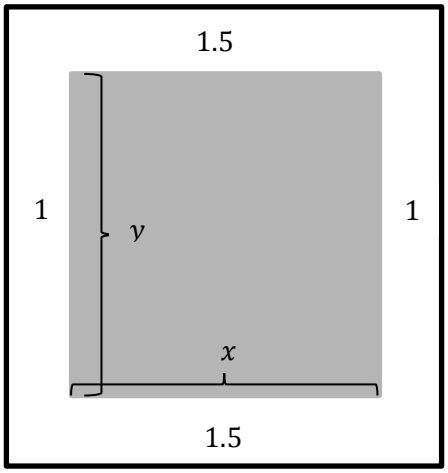
$$V' = 12\pi - \frac{3}{2}\pi r^2$$



Alternatively, you could use the Second Derivative Test to prove that this point is a maximum. To do this, you would need to take the second derivative of V and show that it is negative at $r = \sqrt{8}$.

© LionTutors 2017

Problem 44



$A = (x+2)(y+3)$

$A = (x+2)\left(\frac{24}{x}+3\right)$

$A = 24 + 3x + \frac{48}{x} + 6$

$A = 30 + 3x + 48x^{-1}$

$A' = 3 - 48x^{-2}$

$0 = 3 - \frac{48}{x^2}$

$3 = \frac{48}{x^2}$

$3x^2 = 48$

$x^2 = 16$

$x = 4$

$xy = 24$

$y = \frac{24}{x}$

$y = \frac{24}{4}$

$y = 6$

Page Dimensions:

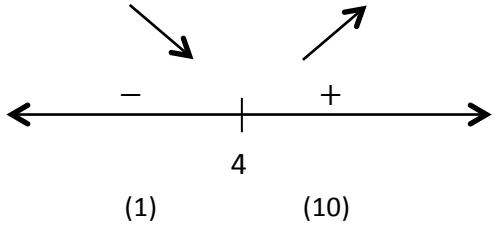
Length: $x + 2 = 4 + 2 = 6$ inches

Width: $y + 3 = 6 + 3 = 9$ inches

© LionTutors 2017

Verification:

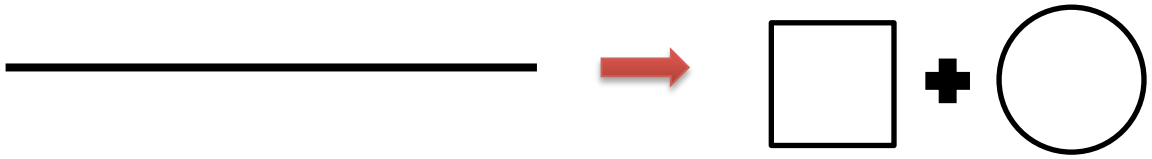
$$A' = 3 - \frac{48}{x^2}$$



Alternatively, you could use the Second Derivative Test to prove that this point is a minimum. To do this, you would need to take the second derivative of A and show that it is positive at $x = 4$.

© LionTutors 2017

Problem 45



$A = x^2 + \pi r^2$

$A = \left(4 - \frac{1}{2}\pi r\right)^2 + \pi r^2$

$A' = 2\left(4 - \frac{1}{2}\pi r\right)\left(-\frac{1}{2}\pi\right) + 2\pi r$

$A' = -\pi\left(4 - \frac{1}{2}\pi r\right) + 2\pi r$

$0 = -4\pi + \frac{1}{2}\pi^2 r + 2\pi r$

$0 = \pi(-4 + \frac{1}{2}\pi r + 2\pi r)$

$0 = -4 + \frac{1}{2}\pi r + 2r$

$4 = \frac{1}{2}\pi r + 2r$

$4 = r\left(\frac{1}{2}\pi + 2\right)$

$r = \frac{4}{\frac{1}{2}\pi + 2} = \frac{8}{\pi + 4}$

$16 = 4x + 2\pi r$

$4x = 16 - 2\pi r$

$x = 4 - \frac{1}{2}\pi r$

$x = 4 - \frac{1}{2}\pi\left(\frac{8}{\pi + 4}\right) = 4 - \frac{4\pi}{\pi + 4}$

$x = 4\left(\frac{\pi + 4}{\pi + 4}\right) - \frac{4\pi}{\pi + 4} = \frac{4\pi + 16 - 4\pi}{\pi + 4}$

$x = \frac{16}{\pi + 4}$

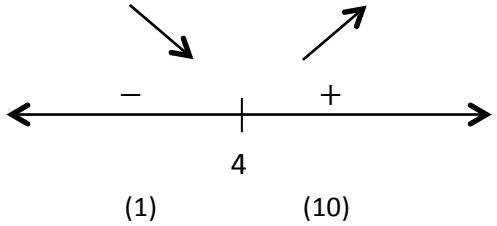
Square: Side of length $x = \frac{16}{\pi + 4}$

Circle: Radius of $r = \frac{8}{\pi + 4}$

© LionTutors 2017

Verification:

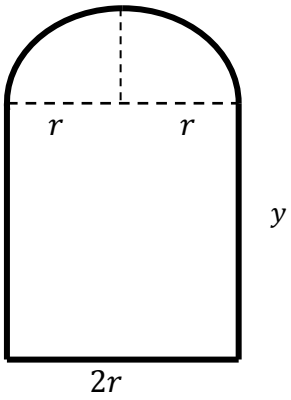
$$A' = -4\pi + \frac{1}{2}\pi^2 r^2 + 2\pi r$$



Alternatively, you could use the Second Derivative Test to prove that this point is a minimum. To do this, you would need to take the second derivative of A and show that it is positive at $r = 4$.

© LionTutors 2017

Problem 46



Before you begin the problem, it should be noted that the question is asking you to solve for r. This means we should be sure to leave the area equation with $r's$ in it. This will prevent us from having to back and solve for r in the end.

$$A = \frac{1}{2}\pi r^2 + 2ry$$
$$A = \frac{1}{2}\pi r^2 + 2r\left(\frac{28-\pi r-2r}{2}\right)$$
$$A = \frac{1}{2}\pi r^2 + r(28-\pi r-2r)$$
$$A = \frac{1}{2}\pi r^2 + 28r - \pi r^2 - 2r^2$$
$$A' = 2\frac{1}{2}\pi r + 28 - 2\pi r - 4r$$
$$A' = \pi r + 28 - 2\pi r - 4r$$
$$0 = 28 - \pi r - 4r$$
$$\pi r + 4r = 28$$
$$r(\pi + 4) = 28$$
$$r = \frac{28}{\pi + 4}$$

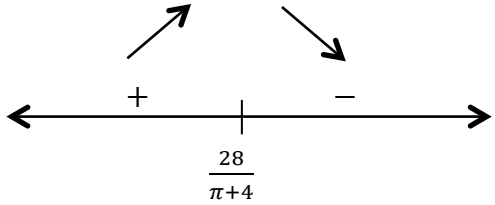
When finding the equation for the perimeter of the window, keep in the mind that the window perimeter is made up half the circumference of a circle and 3 sides of a rectangle. The key to this problem is you need to be able to see that the bottom side of the rectangle is equal to the diameter of the semicircle (also called twice the radius). So instead of calling the bottom side of the rectangle x, we label it as $2r$

$$28 = \frac{1}{2}(2\pi r) + 2y + 2r$$
$$28 = \pi r + 2y + 2r$$
$$2y = 28 - \pi r - 2r$$
$$y = \frac{28-\pi r-2r}{2}$$

© LionTutors 2017

Verification

$A' = 28 - \pi r - 4r$



Alternatively, you could use the Second Derivative Test to prove that this point is a maximum. To do this, you would need to take the second derivative of A and show that it is negative at $r = \frac{28}{\pi+4}$

© LionTutors 2017

Problem 47

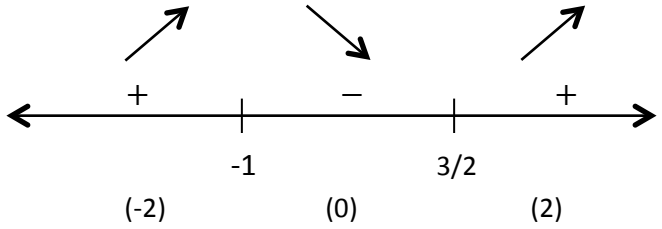
For which x-value(s) on the curve $y = \frac{5}{3}x^4 - \frac{5}{3}x^3 - 15x^2 + 17$ does the **tangent line** (not y) have the **largest slope**?

In this problem we want to maximize the derivative. Remember, to maximize or minimize anything, you take the derivative of that thing and set it equal to 0. So in this case we will be taking the derivative of the derivative (in other words, the second derivative) and setting that equal to 0.

Recall that the first step to any optimization problem is to find the equation of what you want to optimize. In this problem, to find the equation to be optimized, we will take the derivative of y. In order to keep the notation less confusing, I will call this equation T (standing for tangent line). Once you have the equation for T, take the derivative and set it equal to 0.

$$y = \frac{5}{3}x^4 - \frac{5}{3}x^3 - 15x^2 + 17$$
$$T = \frac{20}{3}x^3 - 5x^2 - 30x$$
$$T' = 20x^2 - 10x - 30$$
$$0 = 20x^2 - 10x - 30$$
$$0 = 10(2x^2 - x - 3)$$
$$0 = 10(2x - 3)(x + 1)$$
$$x = 3/2, x = -1$$

At this point, we have two potential answers, $x = 3/2$ and $x = -1$. Now we need to use verify that these are actually maximums. You can use the first derivative test, or the second derivative test to do this. I will use the first derivative test.



According to the Frist Derivative Test, $x = -1$ is a maximum and $x = 3/2$ is a minimum. Since we want to know where the slope is **largest** (maximum), our answer is $x = -1$ only.

© LionTutors 2017